

Therefore also

$$2ty = \frac{2tn^4}{16+m^2} = \frac{n^4}{16+m^2}.$$

By the substitution of these values of y and $2ty$ in (42) and (43), the formulæ (40) are obtained, n in (40) being what we have called $2t$. To find now the root of the equation (41), eliminate m from the equations (40). The result is a sextic equation,

$$\psi(n) = 0.$$

When the coefficients of the quintic (41) are commensurable, the sextic $\psi(n) = 0$ has a commensurable root. Let this be found. Then n is known. Consequently, since $n = 2t$, t is known. Then y is known from (39). Then B is obtained from the second of equations (9), and $B'\sqrt{z}$ from (20). Also

$$u_1 u_4 = g + a\sqrt{z} = a\sqrt{z} = \sqrt{y}.$$

Therefore $u_1 u_4$ is known. Therefore, as in §9, the root of the given quintic is found.

§26. *Third Example.*—As an illustrative example, let

$$x^5 + \frac{625}{4}x + 3750 = 0.$$

Here the equations furnishing the criterion of solvability are

$$\frac{625}{4} = \frac{5n^4(3-m)}{16+m^2},$$

$$3750 = \frac{n^5(22+m)}{16+m^2}.$$

These are satisfied by the values $m = 2$, $n = 5$. Therefore

$$t = \frac{5}{2}.$$

Therefore, by (39),

$$y = \frac{125}{4}.$$

Therefore, by the second of equations (9),

$$B = -\frac{625}{4}.$$

And, by (20),

$$yB'\sqrt{z} = -2(c^2z)(c\sqrt{z}) = -2(t^2y)(t\sqrt{y}),$$

$$\therefore B'\sqrt{z} = -2t^3\sqrt{y} = -\frac{625}{8}\sqrt{5}.$$

$$\therefore B + B'\sqrt{z} = -\frac{625}{4}\left(1 + \frac{\sqrt{5}}{2}\right).$$