where d and t represent respectively the outside diameter and the thickness of the wall of the tube in in. and π the ratio of the circumference to the diameter of a circle. Since $t = 3.464\,r$, where r equals the radius of gyration of cross section of the annulus and of its equivalent columns,

$$\frac{d}{t} = \frac{2l}{3.464\,\pi r} + 1 = 0.1838\,\frac{l}{r} + 1 \ldots\ldots\ldots [3]$$

$$p = \frac{2T}{d} = \frac{2tS}{d} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots [4]$$

where p represents the external fluid pressure in lb. per sq. in., T the total circumferential stress in the wall of the annulus 1 in. long, and S the total circumferential stress per sq. in. of cross section, both being expressed in lb. Also, t and d as before, represent respectively the thickness of the wall and the outside diameter of the tube, in in. Formula [B], using the same notation as before, is

$$p = 86{,}670\,\frac{t}{d} - 1386 \ldots\ldots\ldots\ldots\ldots\ldots [B]$$

By equating the second members of equations [4] and [B] we get

$$\frac{2tS}{d} = 86{,}670\,\frac{t}{d} - 1386$$

from which

$$S = 43{,}335 - 693\,\frac{d}{t} \ldots\ldots\ldots\ldots\ldots\ldots [5]$$

By substituting the value of $\frac{d}{t}$ from equation [3] we get

$$S = 42{,}640 - 127.4\,\frac{l}{r} \ldots\ldots\ldots\ldots\ldots\ldots [K]$$

8. This is a formula for the crippling strength of a column with fixed ends, as derived directly from formula [B] for the collapsing pressures of long tubes that are exposed to external fluid pressures. In this formula, S represents the axial load on the column in lb. per sq. in. of cross section, while $\frac{l}{r}$ represents the slenderness ratio, or the length of column divided by the least radius of gyration of cross section, both being expressed in the same lineal unit.

9. Since formula [B] is applicable to values of thickness divided by outside diameter $\left(\frac{t}{d}\right)$ greater than 0.023, formula [K] should be applicable to values of length of columns divided by least radius of gyration, $\frac{l}{r}$, less than

$$\frac{\frac{\pi}{2}(d-t)}{\frac{t}{3.464}} = 1.732\,\pi\left(\frac{d}{t} - 1\right) = 1.732\,\pi\left(\frac{1}{0.023} - 1\right) = 230$$

Note that formula [G] is tangent to formula [B] at $\frac{t}{d} =$ 0.024, which gives a slenderness ratio $\left(\frac{l}{r}\right)$ at point of tangency of 221, which latter should therefore be the true limiting value of r for formula [K] when used in connection with formula [L] as given below.

10. The Author's Formula [G] for the Collapsing Pressures of Steel Tubes Reduced to an Equivalent Column Formula.—In a manner similar to the above derivation of formula [K] the author's formula [G] has been transformed into an equivalent formula for the crippling strength of long columns or struts. Using the same notation as before collapse formula [G] which is applicable to values of $\frac{t}{d}$ less than 0.024 is

$$p = 50{,}210{,}000\left(\frac{t}{d}\right)^3 \ldots\ldots\ldots\ldots\ldots\ldots [G]$$

and its equivalent column formula, derived in a similar manner, is

$$S = 25{,}105{,}000\left(\frac{1}{0.1838\,\frac{l}{r} + 1}\right)^2 \ldots\ldots\ldots [6]$$

which is applicable to values of $\frac{l}{r}$ greater than 221, as stated above. This somewhat complex formula is represented with sufficient accuracy for all practical purposes by the following simple formula:

$$S = \frac{708{,}000{,}000}{\left(\frac{l}{r}\right)^2} \ldots\ldots\ldots\ldots\ldots\ldots [L]$$

where S as before, represents the axial load on the column in lb. per sq. in. of cross section, and $\frac{l}{r}$ the length divided by least radius of gyration of the column, both being expressed in the same lineal unit. This formula applies only to columns having both ends fixed in direction and for values of $\frac{l}{r}$ greater than 221.

11. **Verification of the Author's Column Formulae [K] and [L] by Comparison with Results of Tests on Columns.**—In order to show that the new column formulae given in this paper are applicable to commercial shapes and annular sections when used as columns with ends fixed in direction, Figs. 6 and 7 were prepared. The only tests on commercial struts and columns with fixed ends known to the writer are those made by James Christie on wrought-iron struts,* and those made at the Watertown Arsenal † in 1909.

12. The average physical properties of the iron in the struts tested by Mr. Christie were

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* Trans. Am. Soc. C.E., 1884, p. 117.

† Proceedings, American Society for Testing Materials, 1909, p. 413.