

$$r_1 = \frac{1}{m} \left(g + \Delta_1 \frac{1}{m} + a_1 \Delta_1 \frac{2}{m} + b_1 \Delta_1 \frac{3}{m} + \dots + e_1 \Delta_1 \frac{m-2}{m} + h_1 \Delta_1 \frac{m-1}{m} \right);$$

where g is rational, and a_1, b_1 , etc., involve only surds subordinate to $\Delta_1 \frac{1}{m}$. §38, 47.

7. The equation $F(x) = 0$ has an auxiliary equation of the $(m - 1)^{\text{th}}$ degree. §35, 52.

8. If the roots of the auxiliary be $\Delta_1, \delta_2, \delta_3, \dots, \delta_{m-1}$, the $m - 1$ expressions in each of the groups

$$\begin{array}{ccc} \Delta_1 \frac{1}{m} \frac{1}{\delta_{m-1}}, & \delta_2 \frac{1}{m} \frac{1}{\delta_{m-2}}, \dots, & \delta_{m-1} \frac{1}{m} \frac{1}{\Delta_1}, \\ \Delta_1 \frac{2}{m} \frac{1}{\delta_{m-2}}, & \delta_2 \frac{2}{m} \frac{1}{\delta_{m-4}}, \dots, & \delta_{m-1} \frac{2}{m} \frac{1}{\delta_2}, \\ \Delta_1 \frac{3}{m} \frac{1}{\delta_{m-3}}, & \delta_2 \frac{3}{m} \frac{1}{\delta_{m-6}}, \dots, & \delta_{m-1} \frac{3}{m} \frac{1}{\delta_3}, \end{array}$$

and so on, are the roots of a rational equation of the $(m - 1)^{\text{th}}$ degree.

The $\frac{m - 1}{2}$ terms

$$\Delta_1 \frac{1}{m} \frac{1}{\delta_{m-1}}, \delta_2 \frac{1}{m} \frac{1}{\delta_{m-2}}, \dots, \delta_{\frac{m-1}{2}} \frac{1}{m} \frac{1}{\delta_{\frac{m+1}{2}}},$$

are the roots of a rational equation of the $\left(\frac{m - 1}{2}\right)^{\text{th}}$ degree. §39, 44, 55.

9. Wider generalization. §45, 57.

10. When the equation $F(x) = 0$ is of the first class, the auxiliary equation of the $(m - 1)^{\text{th}}$ degree is irreducible. §35. Also the roots of the auxiliary are rational functions of the primitive m^{th} root of unity. §36. And, in the particular case when the equation $F(x) = 0$ is the reducing Gaussian equation of the m^{th} degree to the equation $x^n - 1 = 0$, each of the $\frac{m - 1}{2}$ expressions,

$$\Delta_1 \frac{1}{m} \frac{1}{\delta_{m-1}}, \delta_2 \frac{1}{m} \frac{1}{\delta_{m-2}}, \text{ \&c.,}$$

has the rational value n . §41. Numerical verification. §42.