

therefore is equal to the product of $(b+c+a)$ and $(b+c-a)$. Upon substituting the values of b , c , and a the factor $(b+c+a)$ becomes zero.

$$\therefore b^2 + c^2 + 2bc - a^2 = 0,$$

3. Prove that four times the product of two consecutive integers differs from a square integer by unity.

x and $(x+1)$ are consecutive integers.

$4x(x+1)$ = four times their product = $4x^2 + 4x$. It is evident that $4x^2$ and $4x$ are the first terms of the square of a binomial. Add one to complete the square and subtract one to leave the value unchanged and

$$4x^2 + 4x = 4x^2 + 4x + 1 - 1 = (2x+1)^2 - 1 = \text{a square integer} - 1.$$

$\therefore 4x(x+1)$ differs from a square integer by unity.

4. Solve the equation: $\frac{x+1}{x-1} + \frac{x+2}{x-2} = \frac{2x+6}{x-3}$

Divide each numerator by its denominator and the equation becomes:

$$1 + \frac{2}{x-1} + 1 + \frac{4}{x-2} = 2 + \frac{12}{x-3} \quad \therefore \frac{2}{x-1} + \frac{4}{x-2} = \frac{6}{x-3}$$

Clearing of fractions, and reducing, $3x^2 = 5x$. Since x is a factor of both sides of the equation, $x = \text{zero}$ is one value of x which satisfies the equation. It is evident that the other value is $1\frac{2}{3}$. Also since clearing the original of fractions would give an equation of three dimensions in x , one value of x is infinity.

5. (a) Simplify the fraction $\frac{(2x-3y)^2 - (x-2y)^2}{3x-5y}$

The numerator of the fraction is equal to the difference of two squares.

Hence the fraction = $\frac{(2x-3y+x-2y)(2x-3y-x+2y)}{3x-5y}$

$$= \frac{(3x-5y)(x-y)}{3x-5y} = x-y$$

5. (b). Factor $7x^2 + 8xy - 12y^2 - 16x + 28y - 15$.

(1) Each factor must evidently contain x , y and a numerical term.

(2) The x terms and numerical terms of the factors must, on multiplication, give those terms of the expression which do not contain y . The y terms and numerical terms of the factors must, on multiplication, give those terms which do not contain x .

These considerations lead us to (1) reject in succession the terms containing y and x , and factor the remaining trinomials, and (2) determine by trial whether the x and y terms of the factors thus formed give the term containing xy in the product.

$$7x^2 - 16x - 15 = (7x+5)(x-3); -12y^2 + 28y - 15 = (-6y+5)(2y-3)$$

In these factors 5 occurs with $7x$, and also with $-6y$; hence $7x$, $-6y$ and 5 form the factor $(7x-6y+5)$. Similarly, the remaining factor is $(x+2y-3)$. By trial it is evident that the x and y terms of these factors give the term $8xy$ in the product. Hence the given expression can be factored and the factors are $(7x-6y+5)(x+2y-3)$.

6. Find the G. C. M. of $x^3 - 2x^2 + 3x - 2$ and $x^4 - 2x^3 + 5x^2 - 4x + 3$.

Evidently the expressions have no common monomial factor. Perform the operations upon the co-efficients alone. (See Dupuis' Algebra)

$$\text{Let } A = x^3 - 2x^2 + 3x - 2$$

$$A \dots 1 + 0 - 2 + 3 - 2$$

$$B \dots 1 - 2 + 5 - 4 + 3$$

$$B = x^4 - 2x^3 + 5x^2 - 4x + 3$$

$$B \dots 1 - 2 + 5 - 4 + 3$$

$$2B \dots 2 - 4 + 10 - 8 + 6$$