by the equal quantities (1) we get

$$4aba^2 dc^2 = 8ca^4d^2 + a^3c^4$$
$$\therefore 4bcd = 8ad^2 + c^3.$$

6. Solve the equation

$$2x^3 - x^2 - 2x + 1 = 0$$

add x^4 to both sides

$$\therefore x^4 + 2x^3 - x^2 - 2x + 1 = x^4$$
$$x^2 + x - 1 = \pm x^2$$

taking the upper sign $x = 1$

while the lower gives

$$2x^2 + x - 1 = 0$$
$$\therefore x = - 1\tfrac{1}{2}$$

7. Explain how to assert the *two* equalities $a = b$ and $x = y$ in *one* statement.

This may be done in two ways

$$a + x\sqrt{-1} = b + y\sqrt{-1}$$
$$\text{or } (a - b)^2 + (x - y)^2 = 0.$$

Two straight lines are drawn to the base of a triangle from the vertex, one bisecting the vertical angle and the other bisecting the base. Prove that the latter is the greater of the two lines.

Let ABC be the triangle; describe a circle about ABC; bisect BC in D; draw DE at right angles to BC meeting the circle in E, (E and A being on opposite sides of BC) and join EA cutting BC in F. Then since the arcs BE, EC are equal, therefore the angles BAE, CAE are equal, that is, EA bisects the angle BAC. Join AD; then the exterior angle AFD of the triangle DFE is greater than FDE, that is, greater than a right angle, therefore ADF is less than a right angle and therefore AD is greater than AF.

Or thus: AD, DE are together greater than AE, and FE is greater than DE, therefore *a fortiori* AD is greater than AF.

The following brings the problem within the range of Book I. Suppose AC greater than AB; draw AF bisecting BAC, meeting BC in F; draw BH perp. to AF and produce it to meet AC in G, then H is the middle point of BG, and D is the middle point of BC ∴ HD is parallel to GC ∴ AH, CD join the extremities of the parallel lines AC, HD towards the same parts ∴ AH, CD meet only when produced ∴ the point F is in BD and not in DC. Then since AC is greater than AB ∴ the angle ABC is greater than ACB ∴ AFC, being equal to ABC and half A, is greater than AFB which is equal to ACB and half A ∴ AFD is an obtuse angle and ∴ greater than ADF ∴ &c.

That F falls on the side of D opposite the greater side appears at once from Book VI. prop. 3, for if CA is greater than AB, CF must be greater than FB.

If the parallelogram of which AB, AC are adjacent sides be completed, then AD produced will be a diagonal, and therefore nearer the greater side, and consequently the line bisecting BAC will be between AD and the shorter side of the triangle.

GENERAL HINTS ON THE SOLUTIONS OF DEDUCTIONS.

Before proceeding to the actual solution of deductions, it will be well to pay careful attention to two or three points upon which all satisfactory and sound work must be based. And above all others, at the root of all lies *accurate definition*—that is to say, to be able accurately to describe any geometrical figure or operation, and to fully appreciate such definition when heard or read, so as to grasp at once the various and possibly complicated ideas involved in it. For definitions and descriptions may err either through excess or defect—that is to say, facts may be stated which are not to the point, or which are really included in what has been otherwise mentioned; or, on the other hand, some really pertinent fact may be omitted, so that the description only partially defines the figure or operation referred to, for it would equally suit one that differed from it in that very peculiarity. Consider, for instance, the definition of a square: if we described it as a four-sided figure whose angles are right angles, we should err by defect, for we have not stated that it is a rectilineal figure, so that one of the sides might, as far as our definition is concerned, be curved; nor yet have we stated that its sides are equal, both of which