

value as 1284 to 35. A certain volume is composed of equal weights of gold and silver; find how many times more valuable the same volume would be were it composed wholly of gold.

Take value of 1 lb. of silver as unit. If equal volumes of gold and silver are in weight as  $x:1$ , then their values are as  $20x:1$ ;

$$\therefore \text{per question } 20x = \frac{1284}{35}, x = \frac{321}{175}.$$

Let the "certain volume" in question contain  $y$  lbs. of gold and  $y$  of silver, its value being  $21y$ . Replacing  $y$  lbs. of silver by a volume of gold equal to it in bulk, the total value is now  $20\left(y + y \cdot \frac{321}{175}\right)$ , and ratio of latter to former is 1984:735.

10. The volume of a sphere is found by multiplying the cube of the radius by 4.1888; and the area of a circle by multiplying the square of the radius by 3.1416. Find the area of a circle which by rotating about a diameter will describe a sphere whose volume is 1 cubic foot.

Let  $r$  be radius of sphere and circle. Given  $4.1888r^3 = 1728$ , we find  $3.1416r^2 = 173.06 +$  square inches, the area required.

#### EUCLID.

(Usual abbreviations permitted.)

1. A parallelogram is a rectilineal figure whose opposite sides are parallel and whose opposite angles are equal. Show clearly what is deficient and what redundant in this definition.

2. The three angles of a triangle are together equal to two right angles. Prove this, and by its means show how to divide a right angle into three equal parts.

3. Triangles upon the same base and between the same parallels are equal to one another. Prove this, and thence show how to change an irregular four-sided figure into an equal triangle.

4. Given three straight lines, show how to construct a triangle having these lines for sides. Can it always be done? Explain fully.

5. If a straight line be bisected and also cut into two unequal parts, give the relations

existing amongst the segments as expressed in two propositions of the Second Book of Euclid, and prove one of these propositions.

6. Do *one only* of the following:

(a) If  $A, B, C$  be the angular points of a triangle, find an expression for the perpendicular from  $A$  upon the side  $BC$ , in terms of the sides.

(b) If from any point in the circumference of a circle two lines are drawn to the extremities of a diameter, the sum of the squares upon these lines is constant, and the angle contained by these lines is a right angle. [No reference to Euclid, Bk. III.]

7. What proposition of the Second Book would be formed from Euclid II. 12, by bringing the vertex  $A$  down to the point  $D$  in the side  $BC$  produced?

#### First Class Teachers—Grade C.

#### ALGEBRA.

1. Solve the equations

$$(1) \begin{cases} x^2 - xy + x = 9, \\ 4y^2 - 3xy - 2y = -7. \end{cases}$$

$$(2) x^3 - 2x^2y - 3xy^2 = 10 = x^2 - 3xy.$$

Discuss the values of  $x, y, z$  in the equations

$$a = \frac{x-y}{x+y}, \quad b = \frac{y-z}{y+z}, \quad c = \frac{z-x}{z+x}.$$

(1) Add the equations when

$$(x-2y)^2 + (x-2y) = 2; \therefore x-2y = 1 \text{ or } -2. \\ x = 3 \text{ or } -6, \quad y = 1 \text{ or } -\frac{1}{2}, \text{ etc.}$$

$$(2) \text{ We have } \begin{cases} x(x-3y)(x+y) = 10, \\ x(x-3y) = 10, \\ x+y = 1, \quad x = 2 \text{ or } -\frac{4}{3} \\ y = 1 \text{ or } \frac{4}{3} \end{cases}$$

Solving the equations,

$$x(a-1) + y(a+1) = 0, \text{ etc.,}$$

$$x = \frac{0}{2(abc+a+b+c)} = y = z = 0.$$

2. What value of  $x$  will make  $x^2 - 2x$  a minimum?

Apply your method to show that the square is the greatest rectangle that can be inscribed in a given circle.

Let  $x^2 - 2x = \kappa$ ,  $\therefore x = 1 \pm \sqrt{\kappa+1}$ . That  $\kappa$  may be possible,  $\kappa$  cannot be less than  $-1$ , which is therefore the minimum value, for