See Todhunter's Trigonometry.

$$\frac{22}{7}r^2 \sim \left(\frac{7}{9}\cdot 4r^2 + \frac{7}{900}\cdot 4r^2\right) = \frac{r^2}{11575}.$$

8. A cistern is kept constantly supplied with water; supposing it full, it is found that 24 equal taps opened together will empty it in $5\frac{1}{2}$ minutes, and 15 of them will empty it in 13 minutes. How many of them will empty it in 33 minutes?

If 1 represent the capacity of the cistern, the constant supply will be found to be $\frac{14}{11\times 13}$ and the quantity of water emptied by each pipe $=\frac{5}{3\times 11\times 13}$ in 1′. Therefore number of pipes $=$

$$\left(1+\frac{14\times 33}{11\times 13}\right) \div \frac{5\times 33}{3\times 11\times 13} = 11.$$

9. State the rule for finding the characteristic of the logarithm for any number.

Find the number of digits in the integral part of $30^{20}\times 5^{15}\div 2^{11}$, and the number of ciphers between the decimal point and the first significant figure of the decimal representing 3^{-15}.

See Cherriman & Baker's Trigonometry.

$$\log\frac{3^{20}\times 5^{15}}{2^{11}} = 20\log 3 + 15 - 26\log 2.$$
$$= 16\ 715646.$$

∴ number corresponding to this logarithm has 17 digits in its integral part.

$$\log\frac{1}{3^{15}} = -7.1568195.$$

∴ there are seven ciphers between the decimal point and the first significant figure.

[Logarithm of 2 and 3 should have been given for this question.]

10. (1) The base of a triangle is b, and its altitude a, required the distance from the vertex at which a parallel to the base must cut the altitude in order to bisect the triangle.

(2) The perimeter of a right-angled triangle is p, and the radius of the inscribed circle is r; determine the sides of the triangle.

(1) Let x be altitude of triangle to be cut off. Then since similar angles are to one another in duplicate ratio of their homologous sides

$$x^2 : a^2 :: \frac{ab}{4} : \frac{ab}{2}, \quad \therefore x = \frac{a}{\sqrt{2}}.$$

(2) If c be the right angle,

$$c^2 = a^2 + b^2,\ rp = ab;$$
$$\therefore a + b = \sqrt{c^2 + 2rp}$$
$$a - b = \sqrt{c^2 - rp}, \text{ whence, etc.}$$

ALGEBRA (THIRD CLASS).

1. If $\pi = 3.1416$, $a = 5$ inches, and $h = 7$ feet 11 inches, find the value of $2\pi(ah + a^2)$.

$21\frac{5}{8}$ square feet.

2. If $x = .4$, find, correct to one decimal place, the value of $x^8 - 4x^6 - 2x^4 - 52x^2 + 9$.

.613.

3. If $x = a + d$, $y = b + d$, $z = c + d$, prove that $x^2 + y^2 + z^2 - xy + yz + zx$
$= a^2 + b^2 + c^2 - ab - bc - ca.$

Substitute the values of x, y, z on the right-hand side of the equation, and the result will follow.

4. Divide $a^3 + b^3 + c^3 - 3abc$ by $a + b + c$.

$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac).$$

5. Find the factors of

(*a*) $15x^2 - 19xy - 10y^2$.

(*b*) $15(a+b)^2 + 14(a+b)(x+y) - 8(x+y)^2$.

(*c*) $x^3 - x^2y - xy^2 + y^3$.

(*a*) $15x^2 - 19xy - 10y^2 = (5x + 2y)(3x - 5y)$.

(*b*) $15(a+b)^2 + 14(a+b)(x+y) - 8(x+y)^2 = \{5(a+b) - 2(x+y)\}\{3(a+b) + 4(x+y)\}$.

(*c*) $x^3 - x^2y - xy^2 + y^3 = x^2(x-y) - y^2(x-y)$
$= (x+y)(x-y)^2$.

6. Solve

(*a*) $(10x - 11)(11 + 2x) + (5x - 11)(11 + 3x) + (7x - 11)(11 - 5x) = 0$.

(*b*) $(x - 2n + 1)^2 - (2n - 1)^2 = (x - 2n)^2$.

(*a*) $x = \frac{3}{2}$; (*b*) $x = 2n^2$.