$$=\frac{H}{A}-\frac{2A}{A}+1$$
$$=\frac{H}{A}-1.$$

14. $s=(a+l)\frac{n}{2}$

$$=(200-100)\frac{49}{2}=2450.$$

Middle term $=\frac{1}{2}(a+l)=50.$

15. $s=(a+l)\frac{n}{2}$

$$2s=(a+l)n$$
$$=(2l-\overline{n-1}\,d)n$$
$$\therefore dn^2-(2l+d)n+2s=0$$
$$\therefore n=\frac{2l+d\pm\sqrt{(2l+d)^2-8ds}}{2d}$$

(1) In order that $n$ may be positive we must have $2l+d>\sqrt{(2l+d)^2-8ds}$

$\therefore (2l+d)^2>(2l+d)^2-8ds$

$\therefore ds$ must be positive, that is, $d$ and $s$ must have the same sign.

(2) Evidently $(2l+d)^2-8ds$ must be finite and a complete square ($z^2$ suppose.)

(3) $\therefore n=\dfrac{2l+d\pm z}{2d}$

$\therefore 2l+d+z$ and $2l+d-z$ must each be an even multiple of $d$, $\therefore 2l+z$ and $2l-z$ must be each on odd multiple of $d$.

Also since $a=l-\overline{n-1}\,d$

$\therefore$ if $p$, $q$, be the two values of $n$, the sum of the two first terms will equal

$$l-(p-1)d+l-(q-1)d$$
$$=2l+2d-d(p+q)$$
$$=2l+2d-d\frac{2l+d}{d}$$
$$=d.$$

16. In the first series the common ratio is $-\frac{3}{2}$, $\therefore$ sum to $n$ terms

$$=\frac{4}{15}\left\{1-\left(-\frac{3}{2}\right)^n\right\}$$

In second series com. ratio is $-\frac{2}{3}$ $\therefore$ sum to infinity $=\dfrac{\frac{3}{2}}{1+\frac{2}{3}}=\dfrac{9}{10}$

17. $S=\dfrac{r^n-1}{r-1}$

$$\therefore s(r-1)=r^n-1$$
$$=(1+\overline{r-1})^n-1$$
$$=1+n(r-1)+\frac{n(n-1)}{1.2}(r-1)^2-1$$
$$\therefore s_1=n+\frac{n(n-1)}{1.2}(r-1)\quad(1)$$

gives an approximate value for $s$, and a still closer approximation is given by

$$s=n+\frac{n(n-1)}{1.2}(r-1)$$
$$+\frac{n(n-1)(n-2)}{1.2.3}(r-1)^2$$
$$=s_1+(s_1-n)\frac{n-2}{3}(r-1)$$

18. (1). $x=1$.

(2). $x=\pm 1$.

(3). $x=21\frac{8}{11}$, $10\frac{8}{11}$

(4): $\dfrac{x}{y}-\dfrac{y}{x}=b$

$$\therefore x^2-y^2=bxy$$

put $x=ky$

$$\therefore k^2-1=bk$$
$$\therefore k=\frac{b+\sqrt{b^2+4}}{2}$$

again, $xy(x^2+y^2)=a$

$$\therefore ky^2(k^2y^2+y^2)=a$$
$$\therefore y^4=\frac{a}{k^3+k^2}$$
$$\therefore x^4=\frac{ak^3}{k^2+1}$$

These give $x$ and $y$ since $k$ has already been found.

19. $65\frac{5}{11}$ minutes.

20. $(x+a)(y-b)=xy.$

$$(x+c)(y-d)=xy-e$$
$$\therefore x=b\frac{ad+e-cd}{ad-b}$$