The curious thing is that many examination candidates, who show great facility in reducing exceptional equations to quadratics, appear not to have the remotest idea beforehand of the number of solutions to be expected; and that they will very often produce for you, by some fallacious mechanical process, a solution which is none at all. In short, the logic of the subject, which, both educationally and scientifically speaking, is the most important part of it, is wholly neglected. The whole training consists in example-grinding. What should have been merely the help to attain the end has become the end itself. The result is that algebra, as we teach it, is neither an art nor a science, but an ill-digested farrago of rules, whose object is the solution of examination problems."

Perhaps the most important part of this criticism is where the Professor speaks of the omission of any account of what is meant by an integral, rational, algebraical function of x. It is most remarkable that, while the ordinary arithmetical notation deals with one special instance of such a function, where the value of x is 10, and the coefficients must all be positive and less than 10, the elementary books do not generalize this idea, so as to make x arbitrary, and the coefficients positive or negative, and unlimited in value. In arithmetic, the ten, which is the radix of the scale, is not expressed, but is left to be understood; surely, it would be desirable to do the same thing with the x, to go through all the working with detached coefficients, and introduce x only in the final result. A little practice with detached coefficients in working multiplication, division, G.C.M., and square root, would teach a boy more about integral rational functions than he now knows when he goes up to the University. Supposing that the associative, distributive, commutative, and index laws are once stated, and negative quantities recognized, I believe all the rudimentary operations of algebra might be made to fall under the one direction, that they were the same as the corresponding operations in arithmetic, subject to the following alterations:—

(1) There is no carrying, as x is arbitrary, and not a known given quantity. (2) We cannot talk of one expression as being greater or less than another, but only of its being of higher or lower degree. (3) There is never any need of tentative methods to get the first or any other term in the quotient of a division. It is always found by dividing the term first in order of the remainder by the term first in order of the quotient. (4) Certain minor modifications are required in the G.C.M. process. (5) The L.C.M. should be obtained by separation into factors, as it always should be, but more frequently is not, in arithmetic. (6) In addition of fractions, it is often better to add a few together first, and then another, and so on, instead of all at once as in arithmetic.

The notation $f(x)$, as an abbreviation for a function of x, should be explained, and the theorem that if $f(x)$ is divided by $x-a$, the remainder is $f(a)$, should be demonstrated. This proof involves nothing but first principles; it may come immediately after the first four rules have been practised: it is of the utmost use in all the after part of Algebra; and yet, owing to our miserable plan of regulating our order by tradition instead of reason, many a student never hears anything about this theorem till he has long since passed those parts of the subject where it would have been of of the most use to him.

After making acquaintance with $f(x)$, the pupil might be introduced to $f(xy)$ and $f(xyz)$. The various forms of all these functions, whether homo-