APPROXIMATE CALCULATIONS OF TRANSMISSION LINES WITH DISTRIBUTED CAPACITY.

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THE recent adoption of voltages of 110,000 and higher for power transmission, somewhat complicates the computations of transmission lines, due to the effect of capacity which can no longer be neglected at such high voltages. During recent years several articles have been published, giving the exact mathematical treatment of transmission lines with distributed capacity, but these usually involve the use of hyperbolic functions, and the formulas are too complex to be of any practical value.

Dr. Steinmetz, in his discussion of Percy H. Thomas' paper on "Output and Regulation of Long Distance Transmission Lines," (A.I.E.E., 1909, Volume 1, page 712), gives the exact formulas on transmission lines with distributed capacity, in a very much simplified form; while in the June issue of the General Electrical Review is an article by Mr. F. W. Peek, Jr., in which the exact formulas of Steinmetz are still further simplified. However, in using Peek's method it is necessary to perform several multiplications of complex variables. Anyone who does not use complex variables frequently is liable to be out of practice and make mistakes in signs.

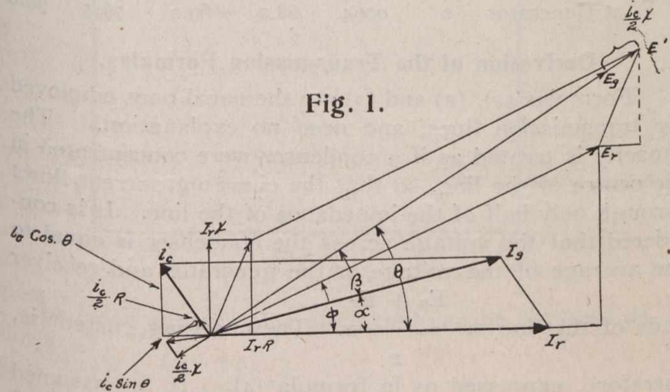

Fig. 1.

Mr. Peek, in his article, works out some examples on a 120,000-volt transmission line 130 miles long, and in his calculations he adds the resistance and reactance of the step-up and stepdown transformers to the resistance and reactance of the line. Since in the transmission line the resistance and reactance are distributed while in the transformers they are concentrated, the formulas, when used in this manner, are therefore not exact, after all. It is also worthy of note that the charging current does not flow through the impedance of the step-down transformers, as assumed by Mr. Peek. In view of the above, there is nothing gained by the use of the exact formulas, if approximate formulas can be found that will give fairly close results. Approximate formulas based on the assumption that the capacity of the transmission line is concentrated at the centre will give results agreeing very closely with Peek's, more exact formulas, though it may sometimes be necessary to calculate some of the values twice, the first calculation being approximate, and the second more accurate.

Below are given some formulas which the writer has found to agree very closely with the exact formulas. Several calculations have been made on Mr. Peek's transmission line so that the values obtained can be compared with those obtained in his article.

Symbols.

Let E_r = receiver voltage between lines,
Let E_g = generator voltage between lines,
Let I_r = current at receiver end of line,
Let I_g = current at generator end of line,
Let i_o = charging current at generator end of line,
x and R = three phase reactance and resistance = $\sqrt{3}$ times the value per log,
Let ϕ = power factor angle of receiver circuit,
Let β = power factor angle at generator,
Let θ = angle between generator voltage, E_g, and receiver current I_r,
Let α = angle between generator current, I_g, and receiver current I_r,
Let c = capacity of line to neutral,
Let l = distance in miles that power is transmitted,
Let d = distance apart of wires, centre to centre in inches,
Let r = radius of wire, in inches,
Let cm = area of conductor in circular miles,
Let $\sim$ = frequency in cycles per second.

Formulas.

(1.) $R = 56{,}000 \frac{l}{cm}$ for copper and $89{,}000 \frac{z}{cm}$ for 63% conductivity aluminium.

(2.) $X = \sqrt{3}\, 2\pi \sim l \left(.00074 \log_{10} \frac{d}{r} + .00008\right)$.

(3.) $C = \frac{.0194}{\log_{10} d} l$.

(4.) $i_o = \frac{2\pi \sim C}{\sqrt{3}\, 10^6} \times \frac{E_g + E_r}{2}$.

(5.) $E_g^1 = \sqrt{(E_r \cos\phi + I_r R)^2 + (E_r \sin\phi + I_r x)^2}$.

(6.) $E_g = \sqrt{(E_r \cos\phi + I_r R)^2 + (E_r \sin\phi + I_r x)^2} - \frac{i_o}{2} x$

$= E_g^1 - \frac{i_o}{2} x$.

(7.) $\sin\theta = \frac{E_r \sin\phi + I_r x}{E_g^1}$.

(8.) $\cos\theta = \frac{E_r \cos\phi + I_r R}{E_g^1}$.

(9.) $I_g = \sqrt{(I_r - i_o \sin\theta)^2 + (i_o \cos\theta)^2}$.

(10.) $\sin\alpha = \frac{i_o \cos\theta}{I_g}$.

(11.) $\beta = \theta - \alpha$

(12a.) Loss in line $= [I_r^2 + I_r (I_g - I_r) + \frac{1}{3} (I_g - I_r)^2] \sqrt{3} R$, when I_g is greater than I_r;

(12b.) or $[I_g^2 + I_g (I_r - I_g) + \frac{1}{3} (I_r - I_g)^2] \sqrt{3} R$ when I_g is less than I_r.

Line Constants of Mr. Peek's Line.

$R = \sqrt{3} \times 39.5 = 68.2$ ohms.
$X = \sqrt{3} \times 182.4 = 316.$ ohms.
$\cos\phi = .85$, $\sin\phi = .53$.
$E_r = 120{,}000$ volts.
$C = 1.735$ M.F.

Check on 5,500 kw. Load at Receiver at 85% Power Factor, I_r = 31.1 Amperes.

From (5) $E_g^1 = \sqrt{(120000 \times .85 + 31.1 \times 68.2)^2 + (120000 \times .53 + 31.1 \times 316)^2}$, $= \sqrt{104125^2 + 73300^2} = 127000$ volts,

As a first approximation assume $E_g = E_r$, and from (4)

$i_o = \frac{2\pi 60 \times 1.735}{\sqrt{3}\, 10^6} \times 120000 = 45.3$ amperes,

From (6) $E_g = 127000 - \frac{45.3}{2} 316 = 127000 - 7150 = 119{,}850$ volts,