

diameter is six units ; to mark out in a field a right-angled triangle whose sides measure ten and four units. . . . We find also in it indications for the measurements of solids, particularly of pyramids, whole and truncated. It appears from the above that the Egyptians had made great progress in practical geometry." As witnessing to the very empirical state of geometry as it existed among the Jews, Babylonians, etc., it is to be noted that they appear to have thought that the circumference of a circle is just three times the length of its diameter. Thus we read that Hiram made for Solomon "a molten sea, ten cubits from the one brim to the other ; it was round all about . . . and a line of thirty cubits did compass it round about." (1 Kings vii. 23.) Even this may be too much to attribute to them ; there is always a danger of reading into statements of this kind more than was originally intended, a danger due to our own vast modern mastery of the science. Possibly Solomon's architect simply found by measurement that the circumference of this particular circle measured in length three times its diameter, without being aware of the general empirical truth that the circumference of every concretely drawn circle bears a fairly fixed ratio to its diameter, much less of the scientific theorem that for all abstractly defined circles this ratio is absolutely fixed (and incommensurable). Incidentally here remark that, unless the idea of a possible numerical dependence of circumference on diameter (or *vice versa*)—the notion, in fact, of a mathematical function—already exists or is suggested by analogy from other experience, there is nothing to urge the mind towards a search for the *precise* measure of this dependence. Here, as elsewhere, we see only what we look for, over and above that which is obvious to all. Now this idea that, in some *definite* way, the two lengths are

numerically related appears to have been born with difficulty. Nor, indeed, is the notion of a mutual numerical dependence common even among modern well-educated people. Many are those who know, and can mechanically apply, the fact that 1,728 cubic inches make one cubic foot, and yet are unaware what dependence this large number has on the fact that twelve inches make one foot. A specific education fails in its due effect in such cases as these, where the bare particular fact is remembered by rote, while the valuable part of the matter (here, the idea of a function) is never assimilated. Such fundamental defects largely characterize elementary education. Egyptian geometry, then, the predecessor of Greek geometrical science, appears to have been practical, approximate, inductive, not scientific, deductive, exact ; in one word, it was *empirical*.

I pass on to Greek geometry. Dr. Allman (in the work above cited) has indicated the precise relation in which Greek geometers stood to their Egyptian predecessors, a relation which appears to have been often misunderstood. It is probable that the influence of J. S. Mill's fallacious treatment of mathematical ideas in his great work on "Logic" is answerable for many of the fallacies and mistakes perpetrated by modern mathematicians in connection with the philosophical basis of their science ; especially is this so in the case of geometry. His constant confusion between *conceptual* thought, which deliberately frames definitions as a basis of deductive reasoning, and *perception*, which is of external objects ; between conceiving, as the result of self-consistent thought, and the quite different conceiving that we call visual imagination ; between the possible in concrete experience and the possible in thought, all lead to the most startling paradoxes. If Kant's famous dictum that "the understand-