

the point 2, and the point 2 approaches towards the point 3, in the ratio of the arcs 12 and 23, or that the point 1 would arrive at the point 2, equally that the point 2 would arrive at the point 3, and by construction the varying intersection a' , would arrive at the point a , at the same time that the varying intersection b' would arrive at the point b . There shall be a point of variation between the points 1 and 2, on the arc 12, and a point of variation between the points 2 and 3, through which by intersections described from B and D, the curve line $a'b'e' \dots n$ shall be varied and shall coincide with its chord line $a'n$.

For let the point 1 arrive at or coincide with the point 2, and the point 2 arrive at or coincide with the point 3—it is evident that the intersections a' , b' , c' , ... n , must also coincide with the intersections a , b , ... n , for the intersection a' must have moved along the arc $a'a'2'$, to the intersection a , and in the same time the intersection b' must have moved along the arc $b'b'3'$ to the point b —hence the curve $a'b' \dots n$ and its chord $a'n$, must coincide with the curve line $ab \dots n$ and its chord an ; but by this variation the point 1, must have passed over a point between 1 and 2, and the point 2, must have passed over a point between 2 and 3, through which were intersections described in the same manner from B and D, by this variation of the points a' and b' these intersections, would be on a straight line passing through the point n ,—for the curve line $a'b' \dots n$ must have changed its convexity to the opposite side of its chord $a'n$ —before it could coincide with the curve line $ab \dots n$ —or that its chord $a'n$ could coincide with the chord an —therefore on this point of change the varied points a' and b' and the point n must be one straight line.

In the same manner it is demonstrated, by supposing the point 2 to vary towards the point 1, and the point 3 to vary towards the point 2,—that the points through which the intersections would make the points a and b vary, till a and b would come to be on a straight line with the point n , must be the same points of variation, as would be by varying 1 towards 2, and 2 towards 3.

In the same manner it is demonstrated, that by making only the points 2' and 3' vary, that is 2' towards the point 3', and 3' towards the point 4'—it is evident that before the intersections a and b , could coincide with the intersections a' and b' —there must be points between 2' and 3' and 3' and 4', through which the intersections described from the points B and D, will bring a and b to be on a straight line passing through the point n ; for the intersection a must move along the arc $ab'2$, and the intersection b must move along the arc $bc'3$, and when the curve line $ab \dots n$ coincides with $b'e' \dots n$, the curve line $nb \dots n$ must have changed its convexity to the opposite side of its chord an , and consequently must have passed points of intersection on a straight line with the point n .

LEMMA 5.

From the point A as a center, describe through the points 1, 2, 3, &c., of the arc BD, the arcs 1T, 2S, and 3R, meeting the arc CD in the points T, S and R. Also through the points 2', 3' and 4', describe the arcs 2'K, 3'O, and 4'Q, meeting the arc CD in the points K, O and Q. Next on the arc BC make the arc B1', equal to the arc B1, B2' equal to the arc B2, B3' equal to the arc B3. Also make the arc B2' equal to B2'—B3' equal to B3 and B4' equal to B4—then from C as a center through the points 1', 2' and 3' describe the arcs 1'T', 2'S' and 3'R', meeting the arc CD in the points T', S' and R'.—Also describe the arcs 2'K, 3'O, and 4'Q through the points 2', 3' and 4'.—Again with the radius AC, through the points B and K, and B and K' describe the arcs BK and BK'—the arc BK intersecting the series of arcs 1K in the points $d, s, r, \dots q, o$, and K, and the arc BK', intersecting the series of arcs 1'K', in the points $t, u, v, \dots q', o'$ and K—then from the point B, as a center, with the distances BK, Bo, and Bq, and from the point K as a center with the distances Kd, Ks and Kr, describe the intersections V, p and q—also from B, with the distances K't, K'u, and K'v, describe the intersections V', p' and q' and draw through Vp and q and through V'p' and q'—the curved lines Vpq and V'p'q'—which by construction are concave to their chords Vu' and Vu'', on the opposite side to that of the curved line $a'b'e' \dots n$. Therefore there shall be an arc of the radius AC, to be described through B, that upon intercepted part of which in the same manner, the intersections V, p, q, &c., may be again described from B and the point of the arc meeting the arc CD—so that those intersec-

FIG. 5.