Using $V_{0,0} = 0$, $q = q^*$ can be iterated in (A2) to yield

$$V_{n,n} = \begin{cases} n \dfrac{(1-rK)^2}{4r} & \text{if } K < 1/r \\ 0 & \text{if } K \geq 1/r \end{cases}$$

These observations permit the recursive solution of (A1) provided optimal current strategies can be determined. Since $V_{n-1,k-1}$ and $V_{n-1,k}$ must be known to determine $V_{n,k}$, an effective iteration sequence is

(n,k) = (2,1), (3,1), (4,1), ..., (3,2), (4,2), ..., (4,3), (5,3), ..., (5,4), ...

But to determine $V_{n,k}$ from (A1), even when $V_{n-1,k-1}$ and $V_{n-1,k}$ are known, requires that optimal strategies $q = q_{n,k}$ and $p = p_{n,k}$ be determined. Because $V_{n,k}$ can be considered to be the payoff of a zero-sum game on the unit square with continuous kernel, optimal (maximin) strategies and a value must exist (Owen, 1982, pp. 67-72).

To find maximin strategies, $V_{n,k}$ can be rewritten as

$$V = p[a + bq - rq^2] + (1-p)[m+q], \qquad 0 < p, q < 1 \qquad \text{(A3)}$$

where $a = V_{n-1,k-1}$, $V_{n-1,k} = m$ and $b = 1-rK$. The following theorem applies:

Theorem: For the game V as in (A3), suppose that a-m>0 and r>0. The equation $(a-m) + (b-1)q - rq^2 = 0$ has exactly one positive root, denoted q_+. Optimal strategies in V are

(i) if $q_+ > 1$, $p^* = 0$, $q^* = 1$

(ii) if $\frac{b}{2r} \leq q_+ < 1$, $p^* = \dfrac{1}{2rq_+ + 1 - b}$, $q^* = q_+$

(iii) if $q_+ < 1$ and $\frac{b}{2r} > q_+$, $p^* = 1$, $q^* = \min \{\frac{b}{2r}, 1\}$

Because the hypotheses of the theorem can be assumed to be satisfied, the determination of all possible values of $V_{n,k}$ is now a matter of iteration as described above, using the Theorem at each step. The results of the theorem were included in a FORTRAN computer program to carry out the recursive solution of (A1). Values such as are shown in Table 1 result from this program. Note that the values of $q_{n,k}$ and $p_{n,k}$ [Table 1(b) and 1(c)] are the optimal strategies in the (n,k) stage. For example, $q_{4,2} = 0.3056$ is E's violation level with 4 time periods and 2 inspections remaining. When only 3 time periods remain, R will have either 1 or 2 inspections left; E will violate accordingly at levels 0.5831 or 0.1447.

Some details illustrating the elasticity calculations reported in the text will be given now. Parameter values r = 0.5 and K = 5.0 are typical, and the case n = 10, k = 3 gives a good illustration. Beginning at $V_{10,3} = 2.40$, a 20% decrease in r to r = 0.4 (holding K constant) leads to $V_{10,3} = 3.42$, a 36% increase. From the same starting point, a 20% increase in r, to r = 0.6, results in $V_{10,3} = 1.87$, a 25% decrease. Thus a change of δ% in r leads to a change