THE MAXIMUM POWER CAPACITY OF A PIPE LINE.

By T. H. Hogg, C.E., Toronto.

In hydro-electric power plant design it is sometimes interesting to know the maximum possible power output of a prospective development which is to have a long feeder conduit. This information is desirable in fixing the diameter of the pipe to be used. To do this accurately entails a study of the economics of the design with reference to the value of the power consumed in friction, the cost of the pipe, transportation conditions, speed-regulation, etc.

The electrical horse-power to be obtained from a given installation is obtained from the formula:

$$P = \frac{62.5\, Q\, h\, E_t\, E_g}{550}$$

where P = electrical horse-power; Q = quantity of water in cu. ft. per sec.; h = net head; E_t = efficiency of turbine; and E_g = efficiency of generator.

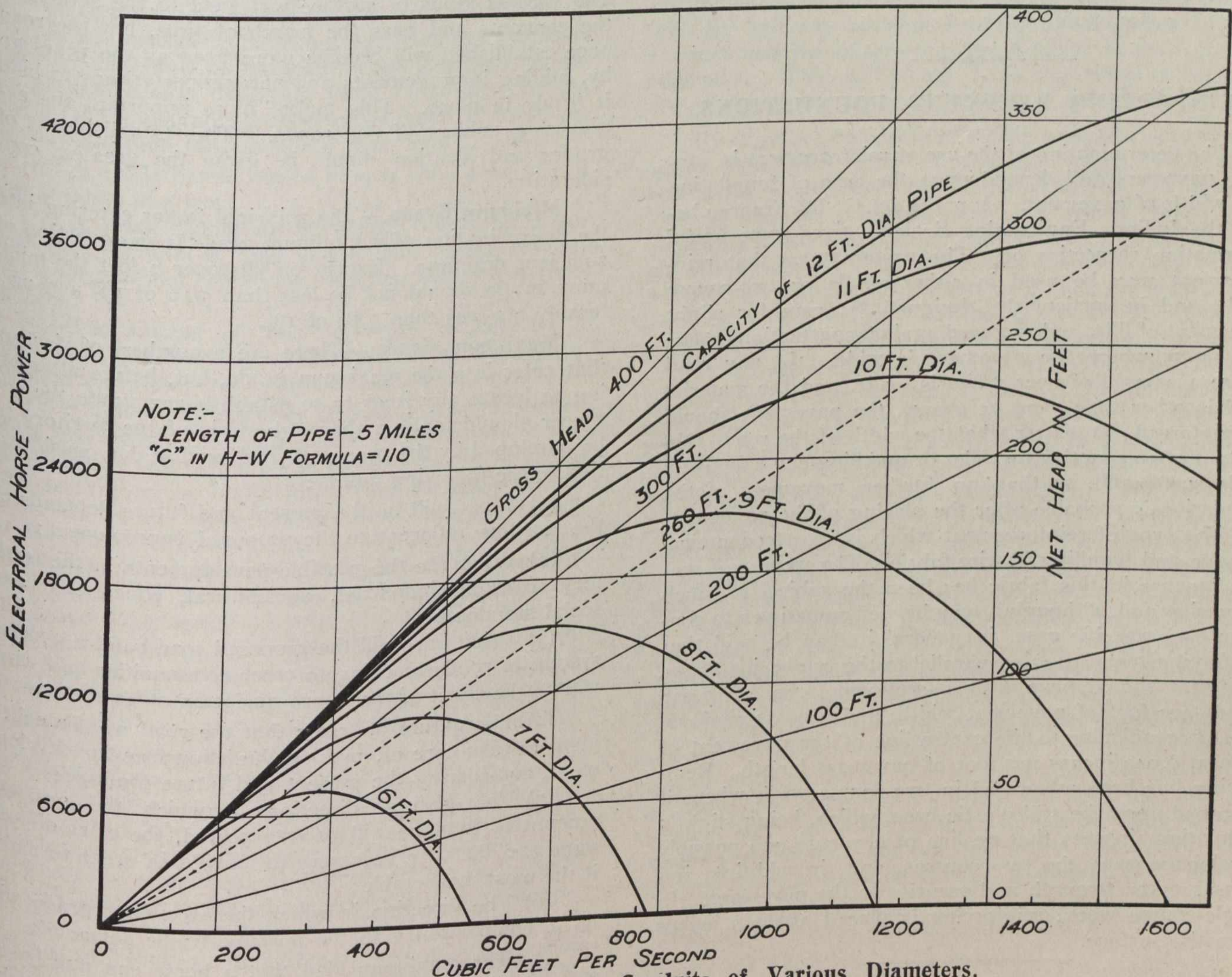
This may be written in the form $P = n\, Q\, h$, where n is a constant, and equal to $\frac{62.5\, E_t\, E_g}{550}$; or $P = n\, A\, v\, h$, where A = area of conduit, and v = velocity in ft. per sec.

Now, $h = H - c\,v^m$, where H = gross head and $c\,v^m = h_f$, or the friction loss in feet. Therefore, $P = n\,A\,v\,h = n\,A\,v\,(H - c\,v^m) = n\,A\,v\,h - n\,A\,c\,v^{m+1}$. In order to get the maximum value of this expression, differentiate with respect to v and equate to zero.

$$\frac{dP}{dv} = n\,A\,H - (m + 1)\,n\,A\,c\,v^m = 0$$

Therefore $H = (m + 1)\,c\,v^m$.

The value of m is a constant usually taken, as a result of experiment, at some value between 1.8 and 2.



Maximum Power Capacities of Conduits of Various Diameters.

Hazen and Williams, in their tables for the losses in pipes, use the value of m equal to 1.85.

Substituting, $H = 2.85\,c\,v^{1.85}$. Now $c\,v^{1.85} = h_f$.

Therefore $c\,v^{1.85} = h_f = \frac{H}{2.85} = \frac{\text{gross head}}{2.85}$. Or, for maximum power in a conduit, the friction loss is equal to the gross head divided by 2.85. If we assume that the losses vary as the square of the velocity this expression will become the gross head divided by 3.

If we use velocities higher than those sufficient to give these losses, the power will decrease as the velocity is increased, until, at the limit, all the available energy in the water is being used up by friction; or when $c\,v^{1.85} = h_f = H$.

The accompanying chart illustrates this very well, as a short study will show. It will be noted that the