$$[(M+M')\,a^2+M k^2+M'k'^2]\frac{d^2x}{dt^2}=(M+M')\,a^2g\sin\alpha. \quad (4)$$

Integrating (3) and (4) twice, and denoting by $s$ and $s'$ the spaces through which the centre moves during the time $t$ in these two cases respectively, we have

$$\frac{s}{s'}=\frac{(M+M')\,a^2+Mk^2+M'k'^2}{(M+M')\,a^2+Mk^2} \quad (5)$$

so that a greater space is described by the sphere which has the fluid than by that which has the solid in its interior.

If the densities of the solid and the fluid are the same, we have from (5), by Art. 233, Ex. 14,

$$\frac{s}{s'}=\frac{7a^5}{7a^5-2a'^5}.$$ (Price's Anal. Mechs., Vol. II, p. 368).

2. A homogeneous sphere rolls down within a rough spherical bowl; it is required to determine the motion.

Let $a$ be the radius of the sphere, and $b$ the radius of the bowl; and let us suppose the sphere to be placed in the bowl at rest. Let $OCQ=\phi$, $QPA=\theta$, $BCO=\alpha$, $\omega=$ the angular velocity of the ball about an axis through its centre $P$, $k=$ the corresponding radius of gyration; $OM=x$, $MP=y$; $m=$ the mass of the ball. Then

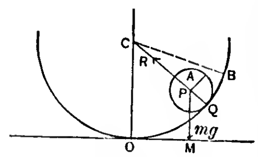

Fig.101

$$m\frac{d^2x}{dt^2}=-R\sin\phi+F\cos\phi; \quad (1)$$

$$m\frac{d^2y}{dt^2}=R\cos\phi+F\sin\phi-mg; \quad (2)$$