tions, supposing the point to be supported, the body to be acted on only by gravity, and the parts of the body to be rigidly connected with the point.

Let the plate be suspended from two points successively, and mark out the vertical line through each point of suspension; then the intersection of these two lines will be the centre of gravity.

Let AB be the lever. Since lever is uniform its weight may be supposed to act at its centre. Let x equal distance of required point from A. Take moments about A.

$$14x = 1\times 1 + 2\times 2 + 3\times 3 + 4\times 4 + 4\times 2\tfrac{1}{2} = 40.$$

$\therefore\ x = \frac{20}{7} = 2\frac{6}{7}$ ft. from A.

5. Explain how to find the relation between the power and weight on a screw. A screw whose pitch is ¼ in. is turned by means of a lever 4 ft. long. Find the power which will raise 15 cwt.

$$P = \frac{\left\{(15\times 112\text{ lbs.})\times \frac{1}{4}\text{ (in.)}\right\}}{4\times 12\times \frac{22}{7}} = 3\tfrac{47}{88}\text{ lbs.}$$

SOLUTIONS

of Algebra Problems (January number) by Iva. E. Martin, St. Catharines.

1. Simplify

(1) $(x-y)^3 + (x+y)^3 + 3(x+y)^2(x-y) + 3(x-y)^2(x+y)$.

(2) $(s-a)^2 + (s-b)^2 + (s-c)^2 + s^2$ when $2s = a+b+c$.

(1) It is evident, from the form of the expression, that it equals

$$\left\{(x-y)+(x+y)\right\}^3 = (2x)^3 = 8x^3.$$

(2) The expression equals

$$(s^2-2as+a^2)+(s^2-2bs+b^2)+(s^2-2cs+c^2)+s^2$$
$$=4s^2 - 2s(a+b+c) + a^2+b^2+c^2$$
$$=4s^2 - 4s^2 + a^2+b^2+c^2 = a^2+b^2+c^2.$$

2. If $xz+yz-xy=2c^2$
$xy+xz-yz=2a^2$
$xy+yz-zx=2b^2$,

prove that $x^2 = \dfrac{(a^2+c^2)(a^2+b^2)}{b^2+c^2}$.

By adding the three equations we obtain

$$xy+yz+zx = 2(a^2+b^2+c^2) \qquad (1)$$

By subtracting each of the given equations from equation (1) we obtain

$$xy=(a^2+b^2)\ (2);\quad yz=(b^2+c^2)\ (3);\quad zx=(a^2+c^2)\ (4);$$

By multiplying equation (2) by equation (4) we obtain $x^2yz=(a^2+c^2)(a^2+b^2)$; and by substituting for yz we obtain

$$x^2(b^2+c^2)=(a^2+c^2)(a^2+b^2).$$
$$\therefore\ x^2 = \frac{(a^2+c^2)(a^2+b^2)}{b^2+c^2}.$$

3. If $\sqrt{ax}+\sqrt{by}+\sqrt{cz}=0$, shew that $a^2x^2+b^2y^2+c^2z^2 = 2(abxy+bcyz+aczx)$.

$$\sqrt{ax}+\sqrt{by}+\sqrt{cz}+0.$$

Multiply both sides of this equation by $(\sqrt{ax}-\sqrt{by}-\sqrt{cz})(\sqrt{ax}+\sqrt{by}-\sqrt{cz})(\sqrt{ax}-\sqrt{by}+\sqrt{cz})$ and it becomes

$$a^2x^2+b^2y^2+c^2z^2-2abxy-2bcyz-2acxz=0.$$
$$\therefore\ a^2x^2+b^2y^2+c^2z^2 = 2(abxy+bcyz+acxz).$$

5. If n be a positive integer, prove that

$$(x+y+z)^{2n+1} - x^{2n+1} - y^{2n+1} - z^{2n+1}$$

is divisible by $(y+z)(z+x)(x+y)$.

By substituting $(-y)$ for x we find that the expression becomes

$$(-y+y+z)^{2n+1} - (-y)^{2n+1} - y^{2n+1} - z^{2n+1} = 0$$

(since $-(-y)^{2n+1} = +y^{2n+1}$; $(2n+1)$ being odd). $\therefore$ $(x+y)$ is a factor of the expression, and, by symmetry, $(y+z)$ and $(z+x)$ are also factors; $\therefore$ the expression, which is of not less than three dimensions, is divisible by $(x+y)(y+z)(z+x)$.

6. (1) If $x^2 - \dfrac{1}{x^2} = y$, express $\dfrac{1+x^4}{1-x^4}$ in terms of y.

(2) If $a = xy^{p-1}$, $b = xy^{q-1}$, $c = xy^{r-1}$, prove that $a^{q-r}\,b^{r-p}\,c^{p-q} = 1$.

(1) $x^2 - \dfrac{1}{x^2} = y \quad (1) \quad \therefore\ x^4 - 1 = x^2y.$

Squaring (1) and adding 4 to each side we obtain

$$x^2 + \frac{1}{x^2} = \pm\sqrt{y^2+4},$$
$$\therefore\ x^4 + 1 = \pm x^2\sqrt{y^2+4}.$$
$$\therefore\ \frac{1+x^4}{1-x^4} = \frac{\pm x^2\sqrt{y^2+4}}{-x^2y} = \frac{\mp\sqrt{y^2+4}}{y}.$$