ARTS DEPARTMENT.

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SOLUTIONS.

The following solutions to Nos. 79, 80 and 82, are given by the proposer, Angus MacMurchy, Univ. Coll.:

79. If the squares of the sides of a triangle are in arithmetical progression, the tangents of the angles are in harmonical progression.

$$a^2 - b^2 = b^2 - c^2.$$

$$\sin^2 A - \sin^2 B = \sin^2 B - \sin^2 C,$$

or

$$\sin(A-B)\sin(A+B) = \sin(B-C)\sin(B+C),$$

$$\therefore \sin(A-B)\sin C = \sin(B-C)\sin A.$$

Expanding and dividing both sides by $\sin A \sin B \sin C$,

$$\cot B - \cot A = \cot C - \cot B,$$

$$\therefore \frac{2}{\tan B} = \frac{1}{\tan A} + \frac{1}{\tan C}.$$

80. If A, B, C be any three quantities; $g_1, g_2, g_3, l_1, l_2, l_3$, the G.C.M.s and L.C.M.s of B and C, C and A, A and B respectively, and if G, L be the G. C. M. and L. C. M. respectively of A, B and C,

$$\frac{L}{G} = \sqrt{\frac{l_1 l_2 l_3}{g_1 g_2 g_3}}.$$

Let a simple factor X occur to power a, b, c in A, B, C respectively. Suppose them to be in descending order of magnitude without excluding cases in which some of them are equal or zero.

Index of X in $\frac{L}{G}$ is $a-c$,

Index of X in $\left(\frac{l_1 l_2 l_3}{g_1 g_2 g_3}\right)^{\frac{1}{2}}$ is $\frac{1}{2}(b+a+a-c-c-b)$
$= a-c.$

The same proof applies to every other factor,

$$\therefore \frac{L}{G} = \left(\frac{l_1 l_2 l_3}{g_1 g_2 g_3}\right)^{\frac{1}{2}}.$$

82. A, B, C is a triangle; a new triangle is formed by joining the feet of the perpendiculars drawn from the angles A, B, C on the opposite sides. Prove that according as one side of the triangle is an A., G. or H. mean between the other two, so is the cosine of one of the semi-angles of the new triangle an A., G. or H. mean between the cosines of the semi-angles of the other two.

Let AD, BE, CF be the perpendiculars meeting in point K, the quadrilaterals $ABDE$, $ACDF$ can each be inscribed in a circle,

$\therefore$ angle EDC = angle BAC = angle FDB.

$$\therefore \frac{1}{2} \text{ angle } FDE = \frac{\pi}{2} - A.$$

$$\therefore \cos \frac{FDE}{2} = \sin A.$$

Now, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$, $\therefore$ cosines of the semi-angles of pedal triangle are proportional to the sides on which their vertices lie; therefore whatever homogeneous relations subsist between the sides of the original triangle must subsist between the cosines of the semi-angles of the pedal triangle.

Solutions to questions 106 and 109, by the proposer, D. F. H. Wilkins, B.A.

106. For the question, see January number, 1880.