

And $u_1 u_4 = \frac{\sqrt{65}}{65}$. Therefore

$$\begin{aligned} x = & \left[-\frac{9+\sqrt{65}}{260} + \sqrt{\left\{ \left(\frac{9+\sqrt{65}}{260} \right)^3 - \left(\frac{\sqrt{65}}{65} \right)^5 \right\}} \right]^{\frac{1}{3}} \\ & + \left[-\frac{9+\sqrt{65}}{260} - \sqrt{\left\{ \left(\frac{9+\sqrt{65}}{260} \right)^3 - \left(\frac{\sqrt{65}}{65} \right)^5 \right\}} \right]^{\frac{1}{3}} \\ & + \left[-\frac{9-\sqrt{65}}{260} + \sqrt{\left\{ \left(\frac{9-\sqrt{65}}{260} \right)^3 + \left(\frac{\sqrt{65}}{65} \right)^5 \right\}} \right]^{\frac{1}{3}} \\ & + \left[-\frac{9-\sqrt{65}}{260} - \sqrt{\left\{ \left(\frac{9-\sqrt{65}}{260} \right)^3 + \left(\frac{\sqrt{65}}{65} \right)^5 \right\}} \right]^{\frac{1}{3}}. \end{aligned}$$

§38. *Twelfth Example.*—Let

$$x^5 + \frac{10x}{13} + \frac{3}{13} = 0.$$

The equations furnishing the criterion of solvability are

$$\begin{aligned} \frac{10}{13} &= \frac{5n^4(3-m)}{16+m^3}, \\ \frac{3}{13} &= \frac{n^5(22+m)}{16+m^2}, \end{aligned}$$

and they are satisfied by the values $m = -7$, $n = 1$. By §25, $n = 2t$. Therefore $t = \frac{1}{2}$. Therefore, from (11), $y = \frac{1}{65}$. Therefore

$$c\sqrt{z} = i\sqrt{y} = \frac{\sqrt{65}}{130}.$$

And, by (20), $yB'\sqrt{z} = -2(c^2z)(c\sqrt{z})$. Therefore $B'\sqrt{z} = -\frac{\sqrt{65}}{260}$. And, by the second of equations (9), $B = \frac{1}{52}$. Therefore

$$B + B'\sqrt{z} = \frac{5-\sqrt{65}}{260}.$$

And $u_1 u_4 = \frac{\sqrt{65}}{65}$. Therefore

$$\begin{aligned} x_1 = & \left[\frac{5-\sqrt{65}}{260} + \sqrt{\left\{ \left(\frac{5-\sqrt{65}}{260} \right)^2 - \left(\frac{\sqrt{65}}{65} \right)^5 \right\}} \right]^{\frac{1}{3}} \\ & + \left[\frac{5-\sqrt{65}}{260} - \sqrt{\left\{ \left(\frac{5-\sqrt{65}}{260} \right)^2 - \left(\frac{\sqrt{65}}{65} \right)^5 \right\}} \right]^{\frac{1}{3}} \\ & + \left[\frac{5+\sqrt{65}}{260} + \sqrt{\left\{ \left(\frac{5+\sqrt{65}}{260} \right)^2 + \left(\frac{\sqrt{65}}{65} \right)^5 \right\}} \right]^{\frac{1}{3}} \\ & + \left[\frac{5+\sqrt{65}}{260} - \sqrt{\left\{ \left(\frac{5+\sqrt{65}}{260} \right)^2 + \left(\frac{\sqrt{65}}{65} \right)^5 \right\}} \right]^{\frac{1}{3}}. \end{aligned}$$