

$$f(x) = \sqrt{a+x}$$

$$f^0(x) = x$$

$$f^2(x) = \sqrt{a + \sqrt{a+x}}$$

$$f^{-1}(x) = x^2 - a$$

$$\text{for } f^1 f^{-1}(x) = \sqrt{a + x^2 - a} = \sqrt{x^2} = x$$

$$f^{-2}(x) = (x^2 - a)^2 - a$$

$$\dots = \dots$$

$$f^{-n}(x) = \left\{ \dots \left\{ (x^2 - a)^2 - a \right\}^2 - a \right\}^2 - a \left\{ \dots \dots \right\}$$

to n brackets.

Note.—Since writing the above, the invaluable treatise of Professor Boole on Differential Equations has been published. In his XVIth chapter there are a few remarks on *inverse forms*, which seem to bear out what has been said on their proper interpretation. He writes, commenting on the index laws as applied to functions: "All that is said above relates to the performance of operations definite in character upon subjects proposed to be given. But an inverse problem is suggested in which it is required to determine, not what will be the result of performing a certain operation upon a given subject, but *upon what subject a certain operation must be performed in order to lead to a given result.*" So below he adds: "If π represent any operation or series of operations possible when their subject is given, and then termed *direct*, and if in the equation $\pi u = v$ the subject u be not given but only the result $= v$ then we may write $u = \pi^{-1} v$. And the problem or enquiry contained in the inverse notation will be answered when we have, by whatever process, so determined the function u as to satisfy $\pi u = v$ or $\pi \pi^{-1} v = v$. By the latter equation the inverse symbol π^{-1} is defined. Thus it is the *office* of the inverse symbol to propose a *question*, not to describe an operation."

If the inverse symbol has an *office*, it is obviously more than a mere convenient notation. The *form* of the above statement may perhaps be open to objection, since when two precisely reverse operations are performed it seems as fair to denote one of them a question as the other. But the view taken of the inverse symbol is the same, whatever be thought of the propriety of this statement.