$(x+y)^7 - x^7 - y^7$
$$= 7xy\,(x+y)\,(x^2+xy+y^2)^2$$

4. Solve $x^3+y^3+z^3 = 1$ (1)
$$x^2+y^2+z^2 = 1 \quad (2)$$
$$x+y+z = 1 \quad (3)$$
From the cube of (3) take (1)

$\therefore x^2y + y^2z + z^2x + xy^2 + yz^2 + zx^2 + 2xyz = 0$ (4)

From the product of (2) and (3) take (1)

$\therefore x^2y + y^2z + z^2x + xy^2 + yz^2 + zx^2 = 0$ (5)

Subtract (5) from (4) $\therefore xyz = 0$

$\therefore$ either $x = 0$, $y = 0$ or $z = 0$

Suppose $z = 0$
$$\therefore x^2+y^2 = 1$$
$$x+y = 1$$
$\therefore xy = 0$ $\therefore$ either x or y is also $= 0$

suppose $y = 0$ then $x = 1$.

$\therefore$ one of the quantities x, y, z is $= 1$ and the other two each $= 0$.

5. Solve $x^2 - yz = a^2$ (1)
$$y^2 - zx = b^2 \quad (2)$$
$$z^2 - xy = c^2 \quad (3)$$
multiply (1) by y, (2) by z, (3) by x and add, then $c^2x + a^2y + b^2z = 0$ (4)

next multiply (1) by z, (2) by x, (3) by y and add, then
$$+ b^2x + c^2y + a^2z = 0 \quad (5)$$
eliminating z from (4) and (5) we have
$$\frac{y}{x} = \frac{b^4 - a^2c^2}{a^4 - b^2c^2} = k \text{ suppose}$$
$$\therefore y = kx$$
eliminating z from (1) and (2) we have
$$x^3 - y^3 = a^2x - b^2y$$
put $y = kx$, then
$$x^2 = \frac{a^2 - kb^2}{1 - k^3}$$
$$= \frac{(a^4 - b^2c^2)^2}{a^6 + b^6 + c^6 - 3a^2b^2c^2}$$

6. Find the sum of the infinite series $ar + (a+ab)r^2 + (a+ab+ab^2)r^3 +$ &c. r and br being each less than unity.

Dividing through by ar we get

$1 + (1+b)r + (1+b+b^2)r^2 +$ &c.

Denote this by S, then Sr
$$= r + (1+b)r^2 + \text{\&c.}$$

Then by subtraction we have
$$S(1-r) = 1 + br + b^2r^2 + \text{\&c.}$$
$$= \frac{1}{1-br}$$
$$\therefore S = \frac{1}{(1-r)(1-br)}$$
$\therefore$ original series
$$= \frac{ar}{(1-r)(1-br)}$$

7. Find the relation between the coefficients of the equation $ax^2 + bx + c = 0$ that one root may be double the other.

The roots are
$$\frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}$$
If, therefore, we put
$$\frac{-b+\sqrt{b^2-4ac}}{2a} = 2\,\frac{-b-\sqrt{b^2-4ac}}{2a}$$
we get $2b^2 = 9ac$ as the required condition.

Or thus :—Let m and $2m$ be the roots of the equation. Then the equation is
$$x^2 - 3mx + 2m^2 = 0$$
$$\therefore 3m = \frac{b}{a},\ 2m^2 = \frac{c}{a}$$
$$\therefore m^2 = \frac{b^2}{9a^2},\ m^2 = \frac{c}{2a}$$
$$\therefore \frac{b^2}{9a^2} = \frac{c}{2a}, \text{ or } 2b^2 = 9ac$$

Solutions to the First-class Euclid Paper given in December number of the MAGAZINE.

6. To construct a rectangle that shall be equal to a given square, and the difference of whose adjacent sides shall be equal to a given line.

Let b denote the side of the given square, and a the difference of the adjacent sides.

Draw a straight line AB equal to a; from A draw AC at right angles to AB and make AC equal $2b$; join BC; from BC cut off BD equal a and bisect DC in E; BE, EC shall be the sides of the required rectangle.