away, to be replaced by universal conceptions. In the first stages of the algebraic methods, material objects may be used to aid the mind in gaining clear notions of the things considered; but no sooner does the mind obtain these notions than it loses sight of the particular and grasps the general. The language employed in analytical investigations is eminently suited to the pure abstractions involved—presenting ideas entirely unconnected with material objects, it is yet capable of representing such objects—universal in power, it is equally applicable to the particular. Every principle in the most elementary of the mathematical sciences is founded on abstraction; every successive stage is reached by a still higher effort of abstraction, while the fundamental principles and ultimate results of the calculus and its applications can be attained only by its highest possible development. This power of mathematics to cultivate the faculty of abstraction establishes one of its most important claims to a high position as a means of intellectual discipline For the faculty of abstraction is undoubtedly connected with the loftiest efforts of the human mind, whether directed to the attainment of moral or intellectual truths. It is the foundation of intellectual and moral philosophy, since the phenomena of the mind, varied, complex and transient as they are, can be carefully observed and truly investigated only by a high degree of abstractive power.

But, in the power of generalization, as well as abstraction, cultivated by the study of mathematics, or is there no generalizations in the sciences, as some assert, because their universal truths are not derived *a posteriori* from experience?

In the opinion of some philosophers, abstraction necessarily implies generalization. Without adopting the view that there can be no abstraction without generalization, since it seems evident that the mind can contemplate certain abstracted qualities of any object, without necessarily establishing a class whose essential marks are given in these qualities, it must be admitted that abstraction is the foundation and necessary condition of all generalization, —Abstraction gives the elements of the concept; generalization moulds them into convenient forms as materials of thought. Hence as mathematics pre-eminently cultivate the power of abstraction, they must qualify the mind for generalization. Admitting that, in obtaining our first conceptions of geometrical truths, "the general is viewed in the particular," the power of abstraction is necessary to give the mind the pure notion which enables it to dispense with sensible objects, and lay the foundation of a pure science. If it be said that the object is still presented to the mind, as a concrete form, by the imagination, I reply that abstraction is necessary to enable the mind to grasp the general as an *a priori* intuition, before the imagination can present the concrete as the representative of the *universal*. And further, in recalling *any* conception to the mind, do we necessarily view all the marks given by abstraction and generalization in the formation of the conception? Do we cognize the general as it is, or grasp it in the particular? It is believed that, though the mind can, by a special exertion of its powers, view the general in its comprehended marks, a particular object is usually recalled as a representative of the class, though with the consciousness that the *individual* possesses many attributes not given in the conception of the *class*. In all the higher geometrical investigations, we are constantly within the confines of the universal—is the universal reached without the generalizing power? In the fundamental propositions and principles there is a classification, and from these the science is unceasingly discovering properties peculiar to distinct classes of conceptions —does not this process of development involve the principle of classification and the power of generalization?

Generalization is also a characteristic feature in Analytical Geometry. "Every process,"—to use the language of J. S. Mill—in Universal Geometry "is a practical exercise in the management of wide generalizations, and abstraction of the points of agreement from those of difference among objects of great and confusing diversity, to which the most purely inductive science cannot furnish many superior." If we pursue the synthetic method of investigation, we shall find that every result, though so far general that it includes a multitude of particulars, is relatively particular, and can be shown to be comprehended in results still more general; and hence every step of our progress demands the exercise of the power of generalizing. Investigating, for instance, the equation of any of the conic sections, we obtain a general expression comprehending a great number of truths—proceeding with the investigation of a second, another result is found equally comprehensive and equally general, and thus, for each figure of the entire class; but the results, though exhibiting each the special property of the conic to which it refers, have nevertheless common characteristics which facilitate their combination into a general expression embracing all the results separately deduced from the independent equations; and if we follow the analytic method, a high degree of abstraction is necessary to enable us to clearly comprehend reasonings founded on conceptions so comprehensive.

But is it true that the analytic method employed in Algebra and the higher mathematics, do not cultivate the power in question, because they substitute a sign for a notion and thus relieve the mind from all intellectual effort? I think not. For though it may not be always so necessary in analytical as in geometrical investigation, to keep in view for the purposes of comparison, the results deduced, a high degree of mental effort, aided by accurate discrimination, is required to enable us to select from the many preceding generalizations, and skilfully apply, the principles necessary to effect the desired synthesis. It is true that the analytical methods, from their precise notation, and higher power of generalization, simplify many geometrical investigations—or rather attain, in a comparitively simple manner, results which geometry can give only by long and cumbrous processes—but the utility they thus lose as an invigorator of mind, is more than restored by their wondrous powers of bringing within its grasp, truths which otherwise would be completely unattainable. But in *any* process of abstract reasoning, do we constantly cogitate the general conception in its essential marks? Or do we not rather use "a sign for a notion," by elevating words to the rank of thoughts? Unless we did so, how complicated would be our mental processes, how unsatisfactory their results, since the difficulty of reasoning increases with the abstruseness of the abstractions involved. So it is with the language of the higher analysis. The reasonings are upon abstract conceptions so comprehensive, that the relation between their successive steps cannot be understood without a vigorous intellectual effort. And though arbitrary symbols are used in analysis, the student must have so clear a conception of the things signified, and their complicated relations, that he is constantly prepared to translate into ordinary language, or interpret by geometry, the results deduced, or he certainly cannot be said to *know* the subject of investigation. Does the difficulty of any process of reasoning increase with the degree of abstraction and generalization of the terms employed? Then analytical investigations must demand a very high degree of mental activity, since they employ the most comprehensive generalization, and are capable of representing in a single view processes and results which would require pages of ordinary language for their elucidation.

As before shown, every first principle of arithmetic and ordinary algebra must proceed from abstraction, and every succeeding principle is a generalized result—from the contemplation of particular examples we attain the general, the universal truth. Every student of the science, has at the outset of his course, experienced the difficulty of rising from the particular illustrations to the universal principles, in consequence of the generalizations involved requring a higher effort of abstraction than his comparatively undeveloped powers can easily attain; but from the cultivation this faculty receives by thorough progress in the science, he ultimately comprehends truths involving a higher degree of abstraction, with greater ease than he had mastered its elementary principles. But the generalizations of the higher analysis and geometric methods, demand a pre-eminent degree of mental energy.—The fundamental principles of those sciences are the result of generalization, or reached only by a high degree of abstraction, and as every demonstration is a generalizing of abstract conceptions, or the analysis of the universal into its comprehended elements, thorough progress cannot be made without a constant exercise of the higher faculties of the mind. By methods of investigation essentially geometric, though aided by analysis, Newton effected the solution of the Lunar inequalities—a problem which had mystified the philosophers of all preceding ages — is there *no* generalization in the results which comprehend these complicated movements? By a more extensive application of analysis, the dynamics of the planetary worlds may be represented in a single view,—does the evolution of results so comprehensive involve no generalization? But it seems to me quite unnecessary to enter into a lengthened defence of the utility of mathematical science as an invigorator of the powers

(CONTINUED PAGE 12.)