SURGE TANK PROBLEMS III.

ANALYSIS AND GRAPHICAL REPRESENTATION OF PROBLEMS DUE TO CONDITIONS OF VARYING OUTFLOW TO PARTIAL OR COMPLETE SHUT-DOWN OR OPENING.

By **PROF. FRANZ PRASIL.**

Authorized Translation by E. R. Weinmann and D. R. Cooper, Hydraulic Engineers, New York City.

Case C.—Variable Outflow.

In the following, we consider first the influence of a gradual shut-down and opening of the penstock, and secondly the case when the outflow is variable with respect to time. The movement during the period of a gradual shut-down will be quite different from that after the shut-down. Therefore, we must consider both cases separately.

(1) MOVEMENT DURING THE SHUT-DOWN.—We assume that the closing occurs so that the outflow decreases in direct proportion to the time which is expressed by the formula:

$$q = Q_1\left(1 - \frac{t}{\mathrm{T}}\right) \quad (56); \qquad \mathrm{T} = \text{time for complete shut-down}$$

Therefore,

$$c = c_1\left(1 - \frac{t}{\mathrm{T}}\right) \qquad \frac{dc}{dt} = -\frac{c_1}{\mathrm{T}}$$

Similar to equation (23) we get the special principal equation (57) in the following form:

$$\frac{d^2z}{dt^2} + \frac{1}{T_0}\cdot\frac{dz}{dt} + \frac{z}{T^2} + \frac{c_1}{T_0}\left(1 - \frac{t}{\mathrm{T}}\right) - \frac{c_1}{\mathrm{T}} = o \quad (57)$$

The general integral of this equation may be obtained by means of the theory of linear differential equations of the second order.

The general member is $z_1 = R\,e^{-\frac{t}{2T_0}} \sin\left(\beta + \frac{t}{T_1}\right)$

The particular integral $z_2 = b_1 + b_2.t$ when b_1 and b_2 may be determined by inserting the value of z_2, also $\frac{dz}{dt} = b_2$ and $\frac{d^2z}{dt^2} = o$ in equation 57.

Therefore,

$$\frac{b_2}{T_0} + \frac{b_1}{T^2} + \frac{b_2 t}{T^2} + \frac{c_1}{T_0} - \frac{c_1 t}{T_0\mathrm{T}} - \frac{c_1}{\mathrm{T}} = o \quad (58)$$

As the conditions of this equation must be fulfilled for all values of t the two equations follow:

$$\frac{b_2}{T_0} + \frac{b_1}{T^2} + \frac{c_1}{T_0} - \frac{c_1}{\mathrm{T}} = o; \qquad \frac{b_2}{T^2} - \frac{c_1}{T_0\mathrm{T}} = o \quad (59)$$

As $h_1 = \frac{c_1 T^2}{T_0}$ $\quad b_1 = h_1\left[\frac{T_0}{\mathrm{T}}\left(1 - \frac{T^2}{T_0^2}\right) - 1\right] \quad b_2 = \frac{h_1}{\mathrm{T}}$

The general integral is obtained by the form $z = z_1 + z_2$.

Therefore,

$$z = R.e^{-t/2T_0} \sin(\beta + t/T_1) + b_1 + b_2 t \quad (60)$$

And with differentiation with respect to t $\left(tg\,\gamma = \frac{2\,T_0}{T_1}\right)$

$$s = \frac{R}{T}\,e^{-t/2T_0} \sin(\gamma - \beta - t/T_1) + b_2 \quad (61)$$

In order to obtain the constant for the integration, consider that for $t = o$, $z = z_0 = -h_1$, as in case (A), but that $s = s_0 = o$, because the shut-down occurs gradually. Therefore:

$$R \sin\beta = -h_1 \frac{T_0}{\mathrm{T}}\left[1 - \frac{T^2}{T_0^2}\right]$$

$$R \cos\beta = -h_1 \frac{T_1}{2\mathrm{T}}\left[3 - \frac{T^2}{T_0^2}\right]$$

$$R = h_1 . \frac{T_0}{\mathrm{T}} . \frac{T_1}{T} \quad (62) \qquad tg\,\beta = 2\,\frac{1 - \frac{T^2}{T_0^2}}{3 - \frac{T^2}{T_0^2}} . \frac{T_0}{T_1} \quad (63)$$

For the graphical demonstration of these functions we may separate them into three equations. For instance, the z — function into

$$r = R\,e^{-t/2T_0\,\phi}; \; z_1 = r \sin(\beta + \phi); \; z_2 = b_1 + (b_2 T_1)\,\phi. \quad (64)$$

The value of r determines again a logarithmic spiral with the slope $tg\,\alpha = -\frac{T_1}{2\,T_0}$ with the initial vectors r_0 and β.

z_1 is obtained in the rectangular co-ordinate system by projection from the polar system as in the former cases. z_2 in the same rectangular co-ordinate system is a straight line which intersects the ordinate axis at the distance b_1 from the initial point and whose inclination to the axis of abscissæ is fixed by the direction constant $b_2.T_1$. The algebraic sum of $z_1 + z_2$ gives z. (See Fig. 5.) But only that part of the curve which lies between the values of the abscissæ o and T is of practical importance because at the time T the complete shut-down has already occurred (as we assumed).

(2) MOVEMENT AFTER COMPLETED SHUT-DOWN.—After the shut-down has occurred, the movement of the water