Cor. An equilateral triangle is greater than any other triangle of equal perimeter. For an equilateral triangle, equal in area to the other, has a less perimeter; and therefore one of equal perimeter will have a greater area.

IX. The perimeter of a square is less than that of any other quadrilateral rectilineal figure which is equal to the square.

Let $ABCD$ be any quadrilateral. Join AC. On AC construct an isosceles triangle AFC equal to ABC; then AF, FC are less than AB, BC by VI. Also on the other side of AC construct an isosceles triangle AGC equal to ADC; then AG, GC are less than AD, DC; and the perimeter of $AFCG$ is less than that of $ABCD$ which has an equal area. Again join FG. On FG construct an isosceles triangle FKG equal to FAG; then FK, KG are less than FA, AG. On the other side of FG construct an isosceles triangle FIG equal to FCG; then FI, IG are less than FC, CG. Hence the perimeter of $FIGK$ is less than that of the equal quadrilateral $FCGA$, and therefore than that of the equal quadrilateral $ABCD$. But it may easily be shown that $FIGK$ is a rhombus or a square. But by IV., Cor. 1, a square equal to this rhombus will have a less perimeter. Still less then will the perimeter of this square be than that of the equal quadrilateral $ABCD$.

Cor. The square is greater than any other quadrilateral rectilineal figure of equal perimeter.

Practical Department.

SOME POINTS IN ALGEBRAIC FACTORING.

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(1). Since the square of a binomial is equal to the square of each of the terms together with twice their product, it follows that to factor a trinomial which is a perfect square, we have simply to connect the square root of each of the squares by the sign of the other term, and write the result twice as a factor. From like considerations we may factor an expression which is the square of any polynome; for the square of a polynome is equal to the square of each term together with *twice each term into all the terms that follow it*. Suppose we wish to factor $x^2+y^2+z^2-2xy+2yz-2zx$.

In this case, the fact that we have *three* squares and *three* double products, suggests that the given quantity is the square of a *trinomial* in x, y, z. To find the signs which are to connect these letters, we have merely to notice the signs of the *double products*; thus, the sign which is to connect x and y will be determined by that of the product $-2xy$; the sign which is to connect y and z will be determined by that of the product $+2yz$, &c. Hence, since the $xy-$ product has the sign *minus*, the signs of x and y must be *different*; therefore we have $x-y$ as two terms of the required trinomial. Further, since the $yz-$ product has the sign *plus*, the signs of y and z must be *alike*, but the sign before y is *minus*, therefore the sign before z must be *minus*; hence the required trinomial is $x-y-z$. Similarly may be factored,

$$x^2+y^2+z^2+u^2-2xy-2yz-2zu-2xu+2xz+2yu.$$

(2). The formula which expresses that "the difference of the squares of two quantities is equal to the product of the *sum* and the *difference* of the *quantities*," can be frequently employed in factoring. We shall notice now *one* of the many important cases. Expressions of the form

$$ax^4+bx^2+c$$

can *always* be resolved into real factors; we shall give some examples of one important class, leaving other cases for future consideration. The class referred to is that which includes expressions in which b is less than twice the square root of ac, of which a typical example is

$$x^4+x^2y^2+y^4.$$

To factor this we throw it into the form above referred to, viz., the *difference of two squares*. Now, one of the squares will be that of a binomial whose terms are the square roots of the *squares* in the given expression:—in this case the square root of x^4 and that of y^4; hence we must have

$(x^2+y^2)^2$, to get which x^2y^2 has to be added to the middle term of the given quantity, and of course must also be subtracted from the resulting square. Hence we have

$$(x^2+y^2)^2-x^2y^2, \text{ which gives } x^2+y^2 \pm xy.$$

Since, therefore, we have merely to add to the middle term what will make the expression a perfect square, *i. e.*, what will make it *twice the product* of *the square roots* of the two squares, we derive the following practical *rule* for factoring such expressions:

(1). Take the square roots of the two extreme terms (*i. e.*, the *squares*) and connect them by the proper sign; this gives the first two terms of the required factors.

(2). Subtract the middle term of the given expression from *twice the product* of these two roots, and the square *roots* of the difference will be the third terms of the required factors.

Apply this to a few examples:—

$$9x^4+8x^2y^2+4y^4.$$

Here the square root of the first term is $3x^2$; that of the last is $2y^2$, and therefore the first two terms of the required factors are $3x^2+2y^2$; twice the product of these is $12x^2y^2$, from which subtract the middle term, and there results $9x^2y^2$, the square roots of which are $\pm 3xy$. Hence the factors are

$$3x^2+2y^2 \pm 3xy.$$

The student will observe that, since the square root of y^4 is $+y^2$, or $-y^2$, it may sometimes happen that while the former sign will give *irrational* factors, the latter will give rational factors; for example,

$$x^4-8\tfrac{1}{4}x^2y^2+y^4.$$

Taking the positive root of y^4, we have, by the rule above given, $x^2+y^2 \pm xy\sqrt{10\tfrac{1}{4}}$; but taking $-y^2$, we have $x^2-y^2 \pm \tfrac{5}{2}xy$.

It may be observed further that sometimes *both* signs will give *rational factors*; for example,

$$16x^4-17x^2y^2+y^4.$$

Here taking $+y^2$ we have, by the rule, $4x^2+y^2 \pm 3xy$; and taking $-y^2$ we have $4x^2-y^2 \pm 5xy$.

(3). By the formulas which enable us to factor trinomial expressions, we can find the factors, where such exist, of a quadratic polynome. Convenient rules may be given for the factoring of trinomials, but we shall omit these for the present, and assuming that the student has acquired some facility in resolving trinomials, we shall point out how polynomes may be factored by the application of perfectly familiar principles. For example, factor

$$3x^2-8xy-3y^2+30y-27.$$

We first of all factor the first *three* terms, getting at once by inspection $3x+y$, and $x-3y$. We have thus obtained two terms of each of the required trinomial factors. Now, to find the remaining terms, we must observe the following conditions:

1°. Their product must equal -27.

2°. The *algebraic* sum of the products obtained by multiplying them diagonally into the y's must equal $+30y$.

3°. The sum of the products obtained by multiplying them *diagonally* into the x's must equal o (*i. e.* $=0.x$). We see at once that -9 with the first binomial above found and $+3$ with the second, satisfy the required conditions, and hence the factors are $3x+y-9$, and $x-3y+3$.

As another example, take

$$12x^2-xy-20y^2+8x+41y-20.$$

We have at once for the factors of the first three terms $4x+5y$ and $3x-4y$.