ARTS DEPARTMENT.

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MATHEMATICAL TRIPOS.

1. Convert $\frac{1}{17}, \frac{2}{17}, \cdots \frac{16}{17}$ into circulating decimals, explaining any methods for deriving one case from another and for shortening the work.

2. Resolve into its component factors

$$(a^3+b^3+c^3)xyz$$
$$+(b^2c+c^2a+a^2b)(y^2z+z^2x+x^2y)$$
$$+(bc^2+ca^2+ab^2)(yz^2+zx^2+xy^2)$$
$$+(x^3+y^3+z^3)\,abc+3abcxyz.$$

Shew also that if $x+y+z+w=0$, then

$$wx(w+x)^2+yz(w-x)^2+wy(w+y)^2$$
$$+zx(w-y)^2+wz(w+z)^2+xy(w-z)^2$$
$$+4xyzw=0.$$

3. Solve the equations:

(i.) $\dfrac{3x-2}{5}-\frac{1}{6}(x-\frac{1}{6})=\dfrac{2x}{51}$,

(ii.) $x^3+y^3=b^3,\ xy+a(x+y)=ab$,

(iii.) $x+y+z=x^2+y^2+z^2$
$$=\tfrac{1}{2}(x^3+y^3+z^3)=3.$$

4. Shew how to insert any number of geometrical means between two given numbers.

An A.P., a G.P. and an H.P. have a and b for their first two terms: shew that the $(n+2)^{\text{th}}$ terms will be in G.P. if

$$\frac{b^{2n+2}-a^{2n+2}}{ba(b^{2n}-a^{2n})}=\frac{n+1}{n}.$$

5. Define a logarithm.

Prove that the logarithm of the product or quotient of two quantities is the sum or difference of their logarithms.

If $x_3=\log_{x_1} x_2,\ x_4=\log_{x_2} x_3 \ldots,$
$x_n=\log_{x_{n-2}} x_{n-1},\ x_1=\log_{x_{n-1}} x_n,$
$$x_2=\log_{x_n} x_1,$$
then $$x_1x_2\ldots x_n=1.$$

6. Find the number of Permutations of n things taken r together.

There are n points in a plane, no three of which lie in a straight line. Find how many closed r-sided figures can be formed by joining the points by straight lines.

vii. Define the unit of circular measure. Assuming that it is constant for all circles, shew that the circumferences of circles vary as their radii.

If an arc of ten feet on a circle of eight feet diameter subtend at the centre an angle $143°.\,14'.\,22''$, find the value of π to four decimal places.

viii. Prove geometrically that

(1) $\cos(A-B)=\cos A\cos B+\sin A\sin B$,

(2) $\sin A+\sin B=2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$,

(3) $\tan(45°+A)-\tan(45°-A)=2\tan 2A$.

Simplify

$$\sin\alpha\sin\beta\,\{\operatorname{cosec}\alpha\operatorname{cosec}(\alpha+\beta)$$
$$+\operatorname{cosec}(\alpha+\beta)\operatorname{cosec}(\alpha+2\beta)$$
$$+\operatorname{cosec}(\alpha+2\beta)\operatorname{cosec}(\alpha+3\beta)\}.$$

ix. Shew that

$$\sin\tfrac{1}{2}A+\cos\tfrac{1}{2}A=\pm\sqrt{1+\sin A},$$

and $$\sin\tfrac{1}{2}A-\cos\tfrac{1}{2}A=\pm\sqrt{1-\sin A}.$$

Hence assuming that $\sin A$ is given, prove that one of the corresponding values of $\tan\frac{1}{2}A$ is $\dfrac{1-\cos A}{\sin A}$. Are we entitled to assume from