

certain number of units a certain number of *times*." Without doubt; but, also, in finding *any* product are we not simply repeating a given unit—group a given number of *times*? We have here a simple yet sufficient principle which explains the "mystery" of division, but which the professor abandons in the moment of his need to plunge into a very wilderness of error from which no Moses can deliver him.

3. He objects—after Col. Parker—very strongly to the use of *times* in the sense of repetition because it tends to perplex the learner. "The reader will observe that I say two, four cubic inches. Because two, four cents is the language of the child and needs no explanation, while *times* has to be explained and then often *times* is not understood. . . . Why will almost 50 per cent. of a class say (young pupils) *three times naught are three*." But this idea of *times* enters into every conception of number and if misunderstood, the true idea of number has never entered into thought. The simplest expression of quantity has necessarily these two components: the unit of measure (the "standard" unit), and the *times* (the how many) of this unit of measure. This *how many* is abstract; it really expresses the *ratio* of the units in the quantity to the standard unit. This is the very pith of the conception of number; and abstract though it is, a firm grasp of it is absolutely necessary to any fair mastery of the properties of numbers. The word *times* has been used in this sense for an indefinite period; it is as old as what is true in the new education. Both term and idea are familiar in the child's experience. And if, after passable teaching, the arithmetical meaning of *times* is still a mystery to any child, it is probably because niggard nature has not endowed the

poor thing with the minimum thirty ounces of brain. And if 50 per cent. of any class of children possessed of normal brains fail to comprehend the meaning of *times* in such expressions as four times two are eight, and fully believe that *three times naught are three*—it is conceivable because they have been reduced to a state of mental inanition by the drivel of some new educator. Such unfortunates may be expected to use the cumbrous phrasing, three, "four dollars," thirty-five and a half, *thirty-six and seven-tenths cubic feet* instead of the simpler expressions that have had the sanction of the ages—three *times* four are twelve, etc.

(b) We come now to the more important part of the professor's article—that in which is concretely unfolded his philosophy of division. He begins with a definition of division which violates every rule of definition. However, we cannot expect those to be strong in definition who strenuously maintain that no rules, definitions and stated principles should have place in the text-book of the future. Definitions are troublesome things—they demand at least an approach to accurate thinking. He says:—"Division is dividing a number into a number of equal numbers. As how many four apples in twelve apples? I say *three four apples*. I express it thus: $12 \text{ apples} \div 4 \text{ apples} = 3$ (four apples). Again: how many hats at \$4 each can I buy with \$12? I say as many hats as there are \$4 in \$12, which are *three four dollars*. Once more: I have $\frac{1}{2}$ of a pie; to how many boys can I give $\frac{1}{2}$ pie. In division dividend and divisor must have the same name. . . . Now we have $\frac{1}{2} \div \frac{1}{2} = 1$. Surely not one whole pie but one one-half pie." And when his imaginary antagonist (Doubter) audaciously suggests that the *one* in this case—as well as the quotients in the other examples, is *abstract*,