**38.** **C** is coplanar with **A**, **B**, when

$$\mathbf{C} = m\mathbf{A} + n\mathbf{B}.$$

Writing the four equations of coordinates, and eliminating $m$ and $n$, we get the coplanar equations

$$|\, a_x\ b_y\ c_z \,| = 0$$
$$|\, a_y\ b_z\ c_u \,| = 0.$$

**39.** To find the perpendicular $\mathbf{N}_2$ from the point **B** to the vector **A**.

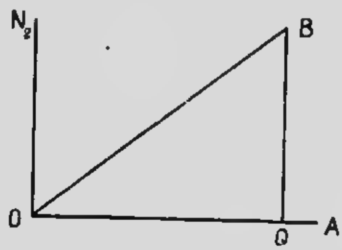


FIG. 10

Let Q B be the positive direction of $\mathbf{N}_2$.

Then $q = \mathrm{OQ} = b\ cas\ \mathbf{AB} = \dfrac{S_{ab}}{a}$.

$$\therefore\ \mathbf{Q} = \frac{S_{ab}}{a_2}\,\mathbf{A}.$$

$$\therefore\ \mathbf{N}_2 = \mathbf{B} - \mathbf{Q} = \begin{vmatrix} S_{aa} & S_{ab} \\ \mathbf{A} & \mathbf{B} \end{vmatrix} \div a^2.$$

$$n^2 = b^2 - q^2 = \begin{vmatrix} S_{aa} & S_{ab} \\ S_{ba} & S_{bb} \end{vmatrix} \div a^2 = \frac{\nu^2}{a^2}.$$

These forms of $\mathbf{N}_2$, $n^2$, **Q**, $q$, are identical in space of four, three and two dimensions, and evidently for space of all dimensions.

In a 2-flat

$$n = \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} \div a.$$

**40.** To find the perpendicular $\mathbf{N}_3$ from **C** to the **AB** plane.

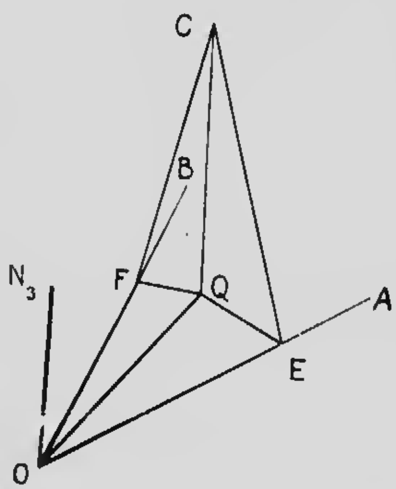


FIG. 11

Let Q C be the positive direction of $\mathbf{N}_3$.
Draw Q E ⊥ **A**, Q F ⊥ **B**, join C F, C E.

Then $\mathrm{OE} = \dfrac{S_{aq}}{a} = \dfrac{S_{ac}}{a}$

$\therefore\ S_{aq} = S_{ac}$ ............ .....(1)

and $\mathrm{OF} = \dfrac{S_{bq}}{b} = \dfrac{S_{bc}}{b}$.

$\therefore\ S_{bq} = S_{bc}$ ............ ......(2)

Since **A**, **B**, $\mathbf{Q}_3$, are coplanar

$|\, a_x\ b_y\ q_z \,| = 0$ ........ ........(3)

$|\, a_y\ b_z\ q_u \,| =$ ............ (4).