

from (1) all the terms not involving a disappear, and the resulting equation can therefore be divided through by x leaving a quadratic.

34. Let the unit of price be such that the cost of a gallon of the mixture is 100; then the selling price is 108; hence the cost of a gallon of the poorer wine is $\frac{100}{11}$ of 108 = $98\frac{2}{11}$; and the cost of a gallon of the better $\frac{100}{10}$ of 108 = $108\frac{2}{5}$. We are therefore required to mix wine costing $98\frac{2}{11}$ per gallon with wine costing $108\frac{2}{5}$ per gallon so as to form a mixture worth 100 per gallon. On every gallon of the poorer wine sold at 100 there is a gain of $1\frac{9}{11}$; therefore on 55 gallons there is a gain of 100. Similarly, on 53 gallons of the better wine sold at 100 per gallon, there is a loss of 100; therefore the mixture must contain 55 gallons of the poorer wine for every 53 of the better.

PROBLEMS.

35. If $ax+by=1$, $cx+dy=1$
 $xy(ad+bc)=1$ shew that

$$\frac{a}{c} + \frac{c}{a} + \frac{b}{d} + \frac{d}{b} = 1$$

36. An oarsman finds that during the first half of the time of rowing over any course he rows at the rate of five miles an hour, and during the second half at the rate of four-and-a-half miles. His course is up and down a stream which flows at the rate of three miles an hour, and he finds that by going down the stream first and up afterwards, it takes him an hour longer to go over the course than by going first up and then down. Find the length of the course. (By Arithmetic.)

37. If $f(x)$ on division by $x-a$ and $x-b$ respectively leaves remainders R, S, show that on division by their product it leaves for remainder

$$\frac{R-S}{a-b}x + \frac{Sa-Rb}{a-b}$$

38. A merchant sells tea, mixed in the ratio of five pounds of green tea to two of black, so as to gain ten per cent. on what the tea cost him. In what proportion must he mix them so as to gain 21 per cent., without increasing the selling price per pound, seven

pounds of green tea being worth nine of black?

39. Solve the equation

$$2x\sqrt{1-x^2}=a(1+x^2)$$

40. An express train leaves Hamilton for Toronto and at the same time a freight train leaves Toronto for Hamilton. They meet at twenty-five minutes past twelve and reach their destinations at ten minutes to one and five minutes past two respectively: find when they started.

41. A and B set out together to walk a certain distance. A walks one half the distance the first day, one-third of the remainder the second day, one-fourth of the remainder the third day, and so on: B walks $\frac{1}{n+1}$ th part of the distance the first day, $\frac{1}{n}$ th of the remainder the second day, $\frac{1}{n-1}$ th of the remainder the third day, and so on: prove that after n days they will be together again. Also find how long it will take each to finish the journey.

42. Prove that $\sin(A-B)\cos(A+B)$
 $+ \sin(B-C)\cos(B+C) + \&c.$
 $+ \sin(N-A)\cos(N+A) = 0$

43. These are $n+1$ vessels which contain each the same quantity (a) of fluid. The contents of the first are distributed equally among the others; then those of the second are distributed in the same way; then those of the third, and so on. Prove that when the last vessel has been thus treated the quantity of fluid contained then in the r th is

$$a \left(1 + \frac{1}{n}\right)^r \left\{ \left(1 + \frac{1}{n}\right)^{n-r+1} - 1 \right\}.$$

44. If a, b, c are in arithmetic progression, and so also x, y, z , while ax, by, cz are in geometric, and $\frac{x}{a}, \frac{y}{b}, \frac{z}{c}$ in harmonic, there will

$$1 - \frac{b^2}{a^2c}, \frac{a}{2b}, \frac{c}{ac} = 1$$

be in harmonic.

45. Solve the equation

$$(x^4+3x^2+34x-37)^{\frac{1}{3}} \\ (x^3-3x^2+34x-37)^{\frac{1}{3}} = 2$$