Thence $m^3 + n^3 = (m+n)^3 - 3mn\,(m+n) = (-{}^{2b}/_a)^3 - 3{}^c/_a\,(-{}^{2b}/_a) = \dfrac{6abc - 8b^3}{a^3}$

(*b*). Form the eqnation whose roots are greater by 2 than the roots of the equation $x^2 - 11x - 17 = 0$.

This is readily done with a little knowledge of the general theory of equations. But, without that, it is easily done as follows.

Let m, n, be the roots of this equation.

Then $m+2$, $n+2$ are the roots of the new equation.

$\therefore\ x^2 - (m+2+n+2)\,x + (m+2)(n+2) = 0$

or $x^2 - (m+n+4)\,x + mn + 2(m+n) + 4 = 0$ is the new equation.

But $m+n = +11$ and $mn = -17$.

Whence, by substitution, $x^2 - 15x + 9 = 0$ is the required equation.

(*c*). Find the square root of $10 + 2\sqrt{21}$.

This may be done by inspection. But systematically, as follows—

Assume $\sqrt{(10 + 2\sqrt{21})} = \sqrt{x} + \sqrt{y}$.

Squaring $x + y + 2\sqrt{xy} = 10 + 2\sqrt{21}$.

By a theorem in algebra, the rational parts of this equation must be equal to one another, and so also must the irrational.

$\therefore\ x + y = 10$, and $4xy = 84$. Whence $x - y = \sqrt{(10^2 - 84)} = 4$.

$\therefore\ x = 7$, $y = 3$ and the root required is $\sqrt{7} + \sqrt{3}$.

6. Fully explain the meaning of x^0, x^{-5}, $x^1/_3$.

The meaning that attaches to these symbols must be deduced from the primary convention in regard to exponents, and cannot be the result of any new convention.

Now it has been agreed that a^n should stand for and represent $1.a.a.a\ldots$ a taken n times as a factor on unity.

Then $a^m = 1.a.a.a\ldots\ a\ m$ times as a factor.

But $\dfrac{a^m}{a^n} = \dfrac{1.a\,a.a\ldots \text{ to } m\ a\text{s}}{1.a.a\,a\ldots \text{ to } n\ a\text{s}} = 1.a.a\ldots(m-n)a\text{s}$

But $1.a.a.a.$ to $(m-n)\,a$s is denoted by a^{m-n} by our convention.

$\therefore\ a^{m-n} = \dfrac{a^m}{a^n}$ Make $n = m$, then $a^0 = 1$.

Make $m = 0$ then $a^{-n} = \dfrac{1}{a^n}$; $\therefore\ x^{-5} = \dfrac{1}{x^5}$

Also, $x^{1/3 + 1/3 + 1/3} = x = x^{1/3} . x^{1/3} . x^{1/3}$.

$\therefore\ x^{1/3}$ signifies that x is to be divided into 3 identically equal factors, and that one of these factors is to be taken. But this is a definition of the cube root of x.