force is

$$\tan Q = \Sigma_h / \Sigma_v \qquad \text{(X)}$$

Factoring out all the $A\gamma$ which affects all the members of (IX), we have

$$\tan Q = \frac{dl r' w^2 / g}{(v^2/2g)\cos Q + H_p + dl - dh - H_s(1-e) + [(r''^2 w^2/2g) - (r'^2 w^2/2g)]} \qquad \text{(XI)}$$

Now equation (XI) is objectionable for one reason at least, because the first two items in the denominator are exceedingly difficult to obtain. This is because the pressure and velocity relations in runners are very complex. But this phase is immediately cleared up when we

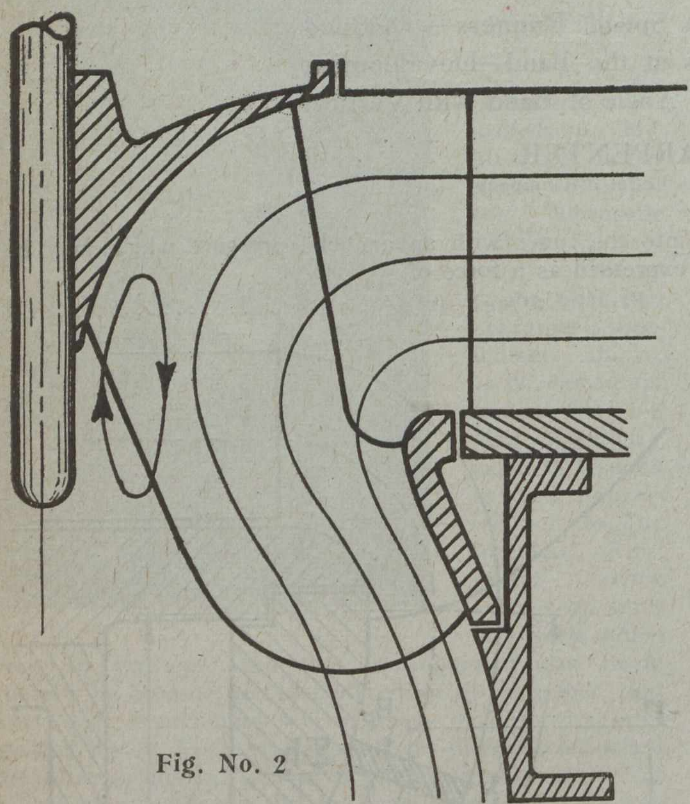

Fig. No. 2

apply a modification of Bernoulli's theorem, in that the sum of the velocity, pressure and friction heads must equal the static head. If we reduce the static head by the friction head, we have the effective head and which as before mentioned takes care of the friction head. Then we can substitute H for the first, second and fifth terms of the denominator of (XI). We then have

$$\tan Q = \frac{dl r' w^2}{gH + dl - dh + (r''^2 w^2/2g) - (r'^2 w^2/2g)} \qquad \text{(XII)}$$

Integrating dl and dh between the limits 1 and 2, we have l and h respectively, and since they are vertical forces and equal when referred to a vertical plane, we can cancel them as they are opposite in sign. This reduces to

$$\tan Q = \frac{l r' w^2}{gH + (r''^2 w^2/2) - (r'^2 w^2/2)} \qquad \text{(XIII)}$$

Reducing the denominator of (XIII) to a common denominator and further simplifying, we have (XIII) in its most convenient form for numerical calculation:—

$$\tan Q = \frac{l r'}{(gH/w^2) + (r''^2 - r'^2)/2} \qquad \text{(XIV)}$$

It will be noticed that the angle increases with an increase in speed, the head remaining the same. Bulging a given runner out at the band will decrease the angle. Operation at part load will bring shock against the band because the apparent head is decreased and which therefore increases the value of Q as obtained from (XIV) if the speed remains the same. Another point in this connection is that the centrifugal force component Σ_h remains the same regardless of gate opening and the other component decreases, thereby increasing Q.

This explains why there are serious secondary cross flows at the upper flange near the lower edge of the buckets (see Fig. 2). Let us take a practical example from a runner design at hand. In this case this runner is under 16 ft. head. $r' = 1.96$ ft., $w = 21$ radians per second, $l = .625$ ft.

Since we know what r'' is because it is a function of r' and Q, we can assume Q at 30° for trial.

$$r'' = r' + l\tan Q = 1.96 + (.625 \times .5774) = 2.32 \text{ ft.}$$

We can now proceed:—

$$\tan Q = \frac{.625 \times 1.96}{(32.2 \times 16/441) + (5.40 - 3.85)/2} = .630,$$

showing that our assumption was too small. In this case the writer takes two-thirds of the difference between .630 and .5774 and adds it to .5774, making a new trial tangent of .612. Again we find r'' as above, this time with $\tan Q = .612$; r'' then is 2.432 ft.

$$\tan Q = \frac{.625 \times 1.96}{(32.2 \times 16/441) + (5.50 - 3.85)/2} = .611,$$

which corresponds to an angle of 31° 30′ in round numbers and is a sufficiently exact figure.

If this same wheel is placed under double the head of the preceding example and the speed changed accordingly, that is in the ratio of the square roots of the heads, the angle Q will not change. Similarly if placed under 8 ft. head the angle will still remain the same. So for any homologous series of runners, the angle Q will be identical, regardless of size.

This angle can be shown to be a function of the specific speed, n_s, as follows: Let all quantities be referred to a one-foot-head basis, and the subscript 1 be added to such quantities.

Taking equation (XIV), H in the denominator can be omitted, as it would not, on a one-foot-head basis, affect the results. The quantity r' is a function of the capacity or power of a runner, in that

$$r' = (M_1 / v_1 \pi)^{1/2} \qquad \text{(XV)}$$

where M_1 is the quantity of water for 1 ft. head, and v_1 is the throat velocity at the same head. Now at 88% efficiency, which is a fair value for present day high speed, high efficiency runners,

$$M = 10N/H \qquad \text{(XVI)}$$

where N is the power.

Substituting (XVI) in (XV), we have

$$r' = (10N_1/v_1\pi)^{1/2} \qquad \text{(XVII)}$$

for 1 ft head.

Inserting (XVII) in (XIV) and reducing the denominator to a common denominator, we have

$$\tan Q = \frac{l w_1^2 (10N_1/v_1\pi)^{1/2}}{g + w_1^2(r''^2 - r'^2)/2} \qquad \text{(XVIII)}$$

Also, w is a function of the speed in revolutions. This is:—

$$w = 360\pi n/60.180 = .1047n \qquad \text{(XIX)}$$

Separating w^2 into $.1047wn$ and bringing N_1 out of the parenthesis of (XVIII), we have

$$\tan Q = \frac{.1047 l w_1 n_1 N_1^{1/2} (10/v_1\pi)^{1/2}}{g + w_1^2(r''^2 - r'^2)/2} \qquad \text{(XX)}$$