SCHOOL WORK.

MATHEMATICS.

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SELECTED PROBLEMS.

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1. A workman is told to make a triangular enclosure of sides 50 41, 21 yards respectively; but having made the first side one yard too long, what length must he make the other two sides in order that they may enclose the prescribed area with the prescribed length of fencing?

2. In the expansion of $(1-x)^{-\frac{1}{x}}$ prove that the sum of the coefficients of the first r terms bears to the coefficient of the rth term the ratio of $1+n(r-1)$ to 1.

3. Sum the following series, each to n terms:

(1) $3+12+33+72+135+$, etc.

(2) $3+14+39+84+155+258+$, etc.

SOLUTIONS.

1. Semiperimeter $=56$,

$$\therefore \text{area} = \sqrt{56\times6\times15\times35}$$

let sides in second case be 51, $41-x$, and $20+x$, then area $=\sqrt{56\times5\times(15+x)(36-x)}$.

$\therefore$ we must have $56\times6\times15\times35=56\times5\times(15+x)(36-x)$, whence $x=5$ or 16, which gives 51, 36, 25, or 51, 25, 36.

2. $$(1-x)^{-\frac{1}{x}}=1+\frac{1}{n}x+\ldots\ldots\frac{\frac{1}{n}\left(\frac{1}{n}+1\right)\ldots\ldots\left(\frac{1}{n}+r-2\right)}{\lfloor r-1}x^{r-1}+\ldots\ldots$$

$$(1-x)^{-1}=1+x+x^2+\ldots\ldots$$

$$\therefore (1-x)^{-1-\frac{1}{x}}=1+\ldots\ldots\ x^{r-1}\left\{1+\frac{1}{n}+x\ldots\frac{\frac{1}{n}\left(\frac{1}{n}+1\right)\ldots\left(\frac{1}{n}+r-2\right)}{\lfloor r-1}\right\}$$

+ etc.,

$\therefore$ the sum of coefficients will be coefficient of x^{r-1} in $\left(1-\frac{1}{x}\right)^{-\left(1+\frac{1}{n}\right)}$

$$=\frac{\left(\frac{1}{n}+1\right)\ \ldots\ldots\ \left(\frac{1}{n}+r-1\right)}{\lfloor r-1}$$

$\therefore$ ratio is $\frac{1}{n}+r-1:\frac{1}{n}$.

3. (1) The nth term is n^3+2n.

(2) The nth term is n^3+n^2+n.

If from any point within an equilateral triangle, perpendiculars be drawn to the three sides respectively, the sum of these perpendiculars will be equal to the altitude of the triangle.

From any point within the equilateral triangle ABC, as D, draw the perpendiculars DE, DF, and DG, and let fall the perpendicular AH, the altitude of the triangle. It is to be proved that $DE+DF+DG=AH$.

Draw the line IJ parallel to BC. Then $DE=HL$, which being deducted from AH, leaves $DF+DG=AL$. Now draw MN parallel to BA: draw NP parallel to DG, and NO parallel to BC or IJ. It then follows from the conditions of the figure that ANO is an equilateral triangle of which NP and AQ are altitudes; or, $AQ=NP=DG$. It now remains to prove that $LQ=DF$. Draw the line NR parallel to QL; then $QL=NR$, the altitude of the triangle DJN. But DF is also an altitude of the same equilateral triangle; then $QL=NR=DF$. Hence $DE+DF+DG=AH$, the altitude of the triangle. Q. E. D.

Two crews, rowing the one 18 miles with the current, and the other 21 miles against it, pass one another, and complete their courses by rowing for 1 and 5 hours more, respectively. Supposing the crews to start at the same time and to row uniformly, find their rates of speed.