The patrimony which a few
Now hold in hugger-mugger in their hand
And all the rest do rob of goods or land.

In Bishop Hall's Satires is this line:

Thwick-thwack and riff-raff! roars he out aloud.

Riff-raff is said by Florio to come from the Italian *ruffola-raffola*; "by hooke or crooke, by pinching and scraping, helter-skelter, higgledie-piggledie." Helter-skelter is supposed by some to have its origin in the Latin *hilariter celeriter*.

(*To be continued.*)

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# UNIVERSITY WORK.

## MATHEMATICS.

ARCHIBALD MACMURCHY, M.A., TORONTO, EDITOR.

### SOLUTION.

By Iva E. Martin, St. Catharines.

(See MONTHLY for March, 1883.)

Let $N=a^{\alpha} b^{\beta} c^{\gamma}$.

All numbers less than $N$ are prime to it, except the series $a$, $2a$, $3a$ .... $N$. (1) $b$, $2b$, $3b$ .... $N$; (2) $c$, $2c$, $3c$ .... $N$; (3) and in these the numbers $ab$, $2ab$ .... $N$; $bc$, $2bc$ .... $N$; $ca$, $2ca$ .... $N$, are repeated; and since in each of the above series of numbers the series $abc$, $2abc$, etc., $N$ occur, we have this series also not prime to $N$.

∴ the sum of the squares of the numbers less than $N$ and prime to it

$$\Sigma N^2 - a^2\,\Sigma\left(\frac{N}{a}\right)^2 - b^2\,\Sigma\left(\frac{N}{b}\right)^2 - c^2\,\Sigma\left(\frac{N}{c}\right)^2 + ab\,\Sigma\left(\frac{N}{ab}\right)^2 + bc\,\Sigma\left(\frac{N}{bc}\right)^2 + ca\,\Sigma\left(\frac{N}{ca}\right)^2 - abc\,\Sigma\left(\frac{N}{abc}\right)^2$$

$$= \left\{\frac{N^3}{3}+\frac{N^2}{2}+\frac{N}{6}\right\} - \left\{\frac{N^3}{3a}+\frac{N^2}{2}+\frac{aN}{6}\right\} - \left\{\frac{N^3}{3b}+\frac{N^2}{2}+\frac{Nb}{6}\right\} - \left\{\frac{N^3}{3c}+\frac{N^2}{2}+\frac{cN}{6}\right\} + \left\{\frac{N^3}{3ab}+\frac{N^2}{2}+\frac{abN}{6}\right\} + \left\{\frac{N^3}{3bc}+\frac{N^2}{2}+\frac{bcN}{6}\right\} + \left\{\frac{N^3}{3ca}+\frac{N^2}{2}+\frac{Nca}{6}\right\} - \left\{\frac{N^3}{3abc}+\frac{N^2}{2}+\frac{abcN}{6}\right\}$$

$$= \frac{N^3}{3}\left\{1-\frac{1}{a}-\frac{1}{b}-\frac{1}{c}+\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ca}-\frac{1}{abc}\right\} + \frac{N}{6}\left\{1-a-b-c+ab+bc+ca-abc\right\}$$

$$= \frac{N^3}{3}\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right).. + \frac{N}{6}\left(1-a\right)\left(1-b\right)\left(1-c\right)....$$

The sum of the cubes and the sum of the fourth powers may be found by substituting in the expressions $\Sigma N^3 - a^3\,\Sigma\left(\frac{N}{a}\right)^3$ – etc., and $\Sigma N^4 - a^4\,\Sigma\left(\frac{N}{a}\right)^4$ – etc., the value of $\Sigma N^3$, $\Sigma\left(\frac{N}{a}\right)^3$ etc.

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## UNIVERSITY OF TORONTO.

ANNUAL EXAMINATIONS: 1883.

*First Year.*

ALGEBRA AND TRIGONOMETRY.

HONORS

*Examiner:* W. FITZGERALD, M.A.

1. Solve,

$$(1)\quad \begin{cases} x^2+xy=65 \\ y^2-xy=24 \end{cases}$$