$n = \frac{2S}{a+l}$ (23). Also $l = a + (n-1)\,d$ (24).

$\therefore\ a = l - (n-1)\,d$ (25); $d = \frac{l-a}{n-1}$ (26); $n = 1 + \frac{l-a}{d}$ (27). Substitute (24) in (20); and,

$S = \frac{n}{2}\left\{2\,a + (n-1)\,d\right\}$ (28); $\therefore\ a = \frac{S}{n} - \frac{(n-1)\,d}{2}$ (29); $d = \frac{2\,(S - an)}{n\,(n-1)}$ (30); and $n = \frac{d - 2a \pm \sqrt{(2\,a-d)^2 + 8\,d\,S}}{2\,d}$ (31). Substitute (25) in 28 and, $S = \left\{2\,l - (n-1)\,d\right\}\frac{n}{2}$ (32);

$\therefore\ d = \frac{2\,(nl - S)}{n\,(n-1)}$ (33); $l = \frac{S}{n} + \frac{(n-1)\,d}{2}$ (34); $n = \frac{d + 2\,l \pm \sqrt{(2\,l+d)^2 - 8\,d\,S}}{2\,d}$ (35). Substitute (27) in (20) and $S = \frac{a+l}{2} + \frac{l^2 - a^2}{2\,d}$ (36); $\therefore\ d = \frac{l^2 - a^2}{2\,S - a - l}$ (37); $a = \frac{d \pm \sqrt{(2\,l+d)^2 - 8\,d\,S}}{2}$ (38); $l = \frac{\sqrt{(2\,a-d)^2 + 8\,d\,S} - d}{2}$ (39).

VI. GEOMETRICAL PROGRESSION.

Let r = common ratio, other symbols as before.

$S = a + ar + ar^2 + ar^3 + ar^4 + \ldots\ldots ar^{n-2} + ar^{n-1}$ (A). Multiplying by r.

$Sr = ar + ar^2 + ar^3 + ar^4 + \ldots\ldots ar^{n-2} + ar^{n-1} + ar^n$ (B). Subtracting (A) from (B).

$Sr - S = ar^n - a$, or, $S(r-1) = a\,(r^n - 1)$ $\therefore\ S = \frac{a(r^n - 1)}{r-1}$ (40); $a = S\left(\frac{r-1}{r^n - 1}\right)$ (41); Also from (A) $l = ar^{n-1}$ (42);

$\therefore\ a = \frac{l}{r^{n-1}}$ (43); $n = 1 + \frac{\log.\ l - \log.\ a}{\log.\ r}$ (44); and