# ARTS DEPARTMENT.

[NOTE.—We continue this month the selection from the Annual Examination Papers of the University of Toronto, for Junior Matriculation; also the selection from the July Examination Papers of the Education Department for First, Second, and Third-Class Teachers, adding solutions to several of the Mathematical papers. ARCHIBALD MACMURCHY, M.A., Math. Ed. C. E. M.]

## UNIVERSITY OF TORONTO EXAMINATIONS, 1879.

### JUNIOR MATRICULATION.

### ALGEBRA.—HONORS.

1. Define a *fraction*, and prove that

$$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd}.$$

Simplify

$$\frac{\frac{1}{1-a}-\frac{1}{1-b}}{\frac{1}{(1-a)b}-\frac{1}{(1-b)a}}\times\frac{\frac{1}{1-b}-\frac{1}{1-c}}{\frac{1}{(1-b)c}-\frac{1}{(1-c)b}}\times\frac{\frac{1}{1-c}-\frac{1}{1-a}}{\frac{1}{(1-c)a}-\frac{1}{(1-a)c}}.$$

2. Describe methods of finding the G.C.M. of two algebraical quantities.

Shew that $(a-b)\ (b-c)\ (c-a)$ is the G.C.M. of $(a+b)\ (a-b)^3+(b+c)\ (b-c)^3+(c+a)\ (c-a)^3$ and $(a-b)\ (a+b)^2+(b-c)\ (b+c)^2+(c-a)\ (c+a)^2$.

Find also the least common multiple of these two quantities.

3. Find the square root and the fourth root of

$$x+x^{-1}-4\sqrt{-1}\ (x^{\frac{1}{2}}-x^{-\frac{1}{2}})-6.$$

If $x^4+2ax^3+bx^2+2cx+d$ is a complete square, prove that

$$a=\frac{c}{\sqrt{d}}=\frac{b-2\sqrt{d}}{a}.$$

4. Find the roots of the equation

$$ax^2+bx+c=o.$$

What do the roots become when (1) $a=o$; (2) $c=o$; (3) $a=o$ and $b=o$?

Prove that a quadratic equation can have only *two* roots.

5. Solve the equations

(1) $\sqrt{2x}+\sqrt{3x}=\sqrt{5}$.

(2) $\left\{(x+l)^2-a^2\right\}\ \left\{(x+l)^2-b^2\right\}$
$=\left\{(x+m)^2-a^2\right\}\times\left\{(x+m)^2-b^2\right\}$

(3) $\frac{1}{x-3}+\frac{3}{x+15}+\frac{1}{x+3}-\frac{5}{x+9}=o.$

(4) $$\left.\begin{aligned}\frac{1}{x}+\frac{1}{z}&=\frac{2}{y}\\ x+z&=\frac{1}{4y}\\ x^2-2yz&=\frac{1}{12}\end{aligned}\right\}$$

6. Find the sum of $n$ terms of an arithmetical series, having given the first term and the common difference.

Find the sum of 32 terms of the *A. P.* whose 5th term is 20, and whose 21st term is 15.

7. Define a harmonic series, and shew how to insert $m$ harmonic means between $a$ and $b$.

If $a$, $2b$ and $c$ be in *H. P.*, then will $a+c$, $a$, and $a-b$, be in *G. P.*, and also will $c+a$, $c$, and $c-b$.

8. Find the number of combinations of $n$ things taken $r$ at a time, and prove that it is the same as the number of combinations of $n$ things taken $n-r$ at a time.

Prove that the number of combinations of $2n$ things taken $n$ at a time is

$$2^n\cdot\frac{1.3.5\ldots.(2n-1)}{1.2.3\ldots.n}.$$

9. Assuming the truth of the Binomial Theorem when the index is a whole number, prove it when the index is a positive fraction.

Write down the fifth term of $(2^3-2)^{-n}$.

Prove that

$$\sqrt[3]{\frac{1}{6}}=\frac{1}{2}+\frac{1}{3.2^3}+\frac{1.4}{1.2}\cdot\frac{1}{3^2.2^5}+\frac{1.4.7}{1.2.3}\cdot\frac{1}{3^3.2^7}+\ldots.$$