CALCULATIONS FOR THE STRENGTH OF ARCHES.

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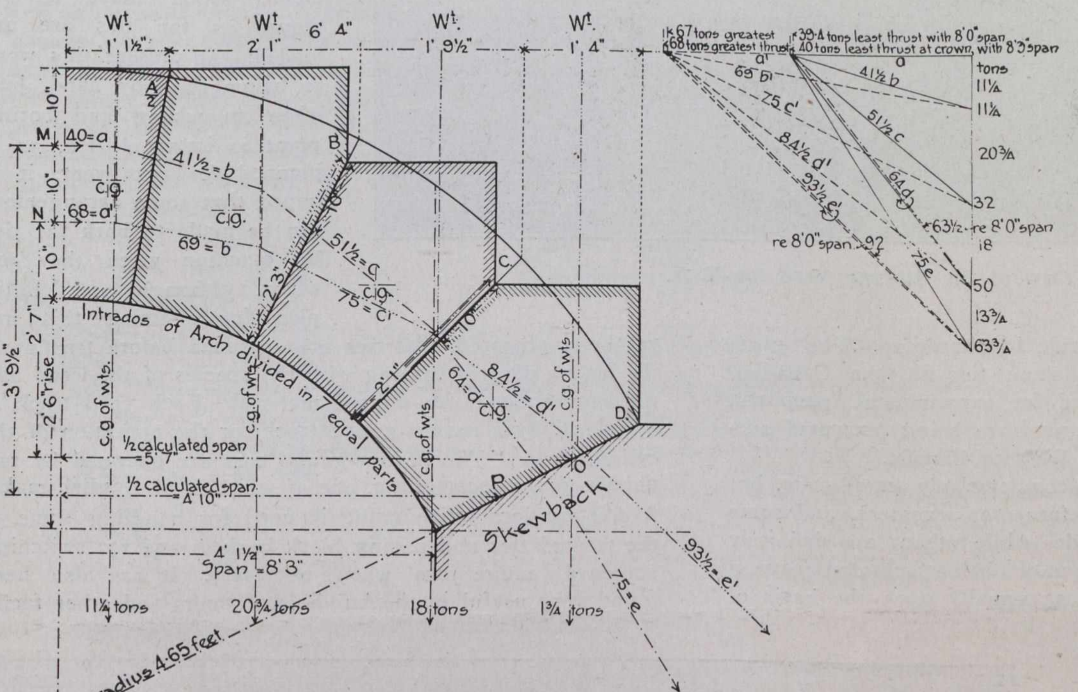
This paper is written with the intention of putting forward clearly a simple and practical method of obtaining the thrust on each voussoir, and finally on the skewback, with accurate results. In most cases it is taken for granted that the reader will understand the reason of each step, and, consequently, no explanation is given. The result is that too often a formula is used without the true principle or knowledge of its construction being known. The example given below was calculated for an engine-house which was built some years ago in the East.



CALCULATIONS FOR STRENGTH OF ARCHES

Span 8' 3". Rise 2' 6".
Sectional area of voussoirs 4' 6" x 2' 6" = 11·25 sq feet.
Stone taken at 125 lbs per cu. ft.
Distributed weight, 9·3 tons per foot run.
Total weight, 127 tons.
Least thrust at centre, $S = \frac{W \times L}{8 \times D} = \frac{127 \times 9\cdot66}{8 \times 3\cdot79}$ = say 40 tons
Greatest thrust at centre, $\frac{127 \times 11\cdot16}{8 \times 2\cdot58}$ = say 68 tons.
Average thrust at centre $\frac{40 + 68}{2} \div 11\cdot25 = 4\cdot8$ tons per sq. ft.
Least thrust on pier = 64 tons = 5·7 tons per sq. ft.
Greatest thrust on pier = $2\overline{)157\tfrac{1}{2}}$ = $93\tfrac{1}{2}$, 78·75 = $2\overline{)14\cdot0}$ = 8·3, 7·0
Average thrust on pier = $\frac{78\cdot75}{11\cdot25}$ = 7·00 tons

Span 8' 0".
Least thrust at centre, $\frac{127 \times 9\cdot41}{8 \times 3\cdot79}$ = 39·4 tons.
Greatest " " " $\frac{127 \times 10\cdot21}{8 \times 2\cdot58}$ = 67 tons
Average thrust at centre becomes 4·7 tons per sq. ft.
Average thrust on pier becomes 7·1 tons per sq. ft.
Allowing for deterioration and imperfections of stone take ⅔ of the sectional area and we get
$\frac{78\cdot75 \times 3}{11\cdot25 \times 2} = \frac{78\cdot75}{7\cdot5}$ = 10·5 tons thrust per sq. ft on pier
In the same way 7·2 tons thrust per sq. ft at centre
For 8' 0" span, thrust at centre becomes 7·1 tons per sq. ft
thrust on pier becomes 10·3 tons per sq. ft.

Calculations and Stress Diagram for Design of Arch.

The method given for finding the least line of resistance and thrusts on the skewback is by graphic strains, which, when drawn carefully to a large scale, is very quick and accurate, and can, of course, be checked mathematically if required. The arch had a clear span of 8′ 3″ with a rise of 2′ 6″ and 4.65′ radius, and was made up of seven voussoirs of equal section for calculation, viz., 4′ 6″ x 2′ 6″. As it is necessary to have the least line of resistance within the middle third M.N.O.P. on the section, divide the arch at its centre and at the skewback into three equal parts, and obtain the spans and vertical distances as on the diagram for the calculation of the least and greatest thrusts at the centre preparatory to constructing the stress diagram. The vertical dotted lines indicate the centres of the distributed weights on each voussoir, and the centre of gravity of each stone is indicated by c. g., whilst the vertical full lines represent the centre of gravity of the total weights, including voussoirs. To obtain the least thrust at centre, the shortest span and greatest depth of the middle third should be taken, and for the greatest thrust the longest span and least depth should be taken.

It will be noticed that each thrust is lettered the same as the voussoir upon which it acts, the only difference being for the greatest thrusts which have a dash over them.

The construction of the stress diagram is as follows:—

Draw to scale the horizontal thrust a, equal to 40 tons as calculated.

From the right-hand end draw the vertical line, composed of the weights in rotation, commencing from the centre of arch.

From the left-hand end draw the lines b, c, d, e to the ends of each line representing the weights. Next, draw to scale the greatest horizontal thrust a′, equal to 68 tons as calculated, and proceed as before and as shown. This completes the resolution of forces, which are built up step by step.

Beginning with the least thrust at the crown on the half section of arch, draw the horizontal thrust a from the point M until it intersects the centre line of weight through voussoir a, and from this point draw the line b parallel to the line b on the stress diagram until it intersects the centre line of weight through voussoir B, and so on to thrust d on skewback, viz., 64 tons, which equals the least thrust. For the greatest thrust at centre, viz., 68 tons, draw the horizontal thrust a′ from the point N until it intersects the centre of weight through voussoir a, and continue as before to thrust e′ on skewback, viz., 93½ tons, which equals the greatest thrust. From these thrusts the average thrust per square foot is obtained on the skewback and centre of arch, as shown in the calculations given below the sketch and stress diagram; and lastly, the thrust per square foot for a sectional area of two-thirds the sectional area of the voussoir, for reasons already explained.

The final results and the course of the line of resistance now obtained show that the stability of the arch is certain, and the profile of the voussoirs may be drawn out.

To do this mark off from the least line of resistance one-third of the depth, viz., 10 inches at the first and second points from the skewbacks, and construct the voussoirs, which, as will be seen, are in proper and uniform proportion to one another. The stability of an arch of this type is easier to obtain than a semi-circular one, as it is most difficult to keep the line of resistance within the middle third, an arch of this form cannot be crippled if properly constructed.