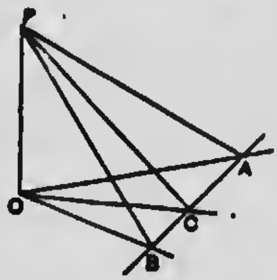
OP is ⊥ to *OA* and *OB*, and *OC* is any line through *O* complanar with *OA* and *OB*. Then *OP* is ⊥ to *OC*.



Proof. Take $OA = OB =$ any convenient length. Join *AB*, cutting *OC* in *C*, and join *PA*, *PB*, *PC*.

The right-angled triangles *POA* and *POB* are congruent, and therefore $PA = PB$. Hence the △ *APB* and *AOB* are each isosceles, and *PC* and *OC* are lines from the vertices to the common base *AB*.

$$\therefore PB^2 - PC^2 = BC \cdot CA = OB^2 - OC^2, \quad \text{(P. Art. 174.)}$$

and $$\therefore PB^2 - OB^2 = PC^2 - OC^2.$$

But *POB* being a ⌉, (hyp.)

$$PB^2 - OB^2 = OP^2 = PC^2 - OC^2.$$

$$\therefore \angle POC \text{ is a } ⌉.$$

Cor. 1. If *O* is fixed while *OA* revolves about *OP* as an axis, *OA* generates a plane to which *OP* is a normal.

Def. A line is perpendicular to a line which it does not meet when a plane containing one of the lines can have the other as a normal.

Cor. 2. A normal to a plane is perpendicular to every line in the plane, and all normals to the same plane are parallel to one another.

Cor. 3. From any point without or within a plane, only one normal can be drawn to the plane.

10. Theorem. Of the line-segments from a point without a plane to the plane: —

1. The shortest is along the normal through the point.