To find the unit shear on vertical or horizontal planes*, consider a slice of the dam, bounded by horizontal planes at $h = 199$ and $h = 200$, the water face and a vertical plane, at a distance, x, from the inner face (Fig. 2), in equilibrium under the water pressure acting horizontally on its left face and the forces exerted by the other parts of the dam on the slice. These forces consist of the uniformly increasing stress, P′, on top, acting down; the uniformly increasing stress, P, on the bottom, acting up; a shear acting on the vertical plane at the right, of average intensity q_1 per square foot; the weight of the body $(x - 0.01)$, besides the horizontal forces to be given later. The vertical component of the water pressure is here neglected, as usual. The origin for x is taken, here and in all subsequent work, at the level, $h = 200$, at the inner face.

For equilibrium, the sum of the vertical components must be zero.

Therefore, $q_1 = (x - 0.01) + P' - P$ .... (2)

To find P′, substitute in Equation 1, $x' = x - 0.02$, $p_2 = 32.22542$, $p_1 - p_2 = 145.22941$, $b = 133.330$, giving $P' = 32.20364\ x + 0.5446238\ x^2 - 0.6442906$. P, $x' = x$, $p_2 = 32.24139$, $p_1 - p_2 = 146.1441$, and $b = 134$; therefore

$$P = 32.24139\ x + 0.5453138\ x^2.$$

Substituting in Equation 2, we derive the average unit shear,

$$q_1 = -0.6542906 - 0.96225\ x - 0.0006900\ x^2 \quad (3)$$

This value of q_1 is strictly correct when $x > 0.02$. It is slightly in error when $0 < x < 0.02$.

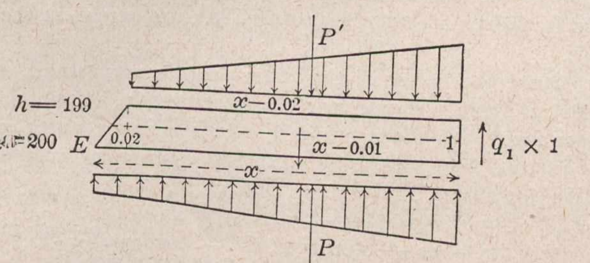

Fig. 2.

A similar investigation holds to obtain the average unit shear q_2 (Fig. 3) on a vertical plane, at a distance, x, from E, extending from the level, $h = 200$, to the level, $h = 201$.

We have, for equilibrium,

$q_2 = (x + 0.01) + P - P''$ .... (4)

We find P″ by substituting in Equation 1, $x' = (x + 0.02)$, $p_2 = 32.25798$, $p_1 - p_2 = 147.05802$, and $b = 134.67$. $P'' = 32.27982\ x + 0.5459941\ x^2 + 0.6453780$. Substituting this, and the value previously found for P, in Equation 4, we derive,

$$q_2 = -0.6353780 - 0.96157\ x - 0.0006803\ x^2 \quad (5)$$

This is strictly correct only when $x > 0$.

The mean, $\frac{1}{2}(q_1 + q_2)$, of these average shears will be assumed as approximately equal to the intensity of shear at the point, G, ($x = $ E G), at the level, $h = 200$. Call q this intensity of shear on a vertical plane at G; therefore,

$$q = -0.6448343 + 0.96191\ x - 0.0006856\ x^2 \quad (6)$$

Checks.—By Appendix (b) and (d), the exact value of q, at either face, $= p \tan. \phi$, where $p =$ vertical unit normal stress at the face and ϕ is the angle the face makes with the vertical. Thus, at the inner face, $q = -32.24139 \times 0.02 = -0.6448278$, whereas Equation 6 gives, for $x = 0$, $q = -0.6448343$.

At the outer face, the exact value is, $178.3855 \times 0.65 = 115.9506$, whereas Equation 6 gives, for $x = 134$, $q = 115.9405$.

A still more searching test can be devised. It is a well-known principle that the intensity of shear at a point on vertical or horizontal planes, is the same [Appendix (a)]. Therefore, regarding Equation 6 as giving the horizontal unit shear, at the level, $h = 200$, where $b = 134$ ft.; the total shear, from face to face, on this level, is,

$$\int_{x=0}^{x=134} q\ dx = 7999.75$$

This should equal the total water pressure down to the same level, $\frac{1}{5}(200)^2 = 8\ 000$. Formula 6 thus gives practically exact results.

In order to find the normal unit stress on a vertical plane, we shall assume that q_1, given by Equation 3, equals the intensity of shear on a vertical or horizontal plane at the point x, at $h = 199.5$; and that q_2, given by Equation 5, gives the shear intensity at x at $h = 200.5$. This evidently supposes that the shear intensity increases uniformly, vertically, from $h = 199$ to $h = 201$.

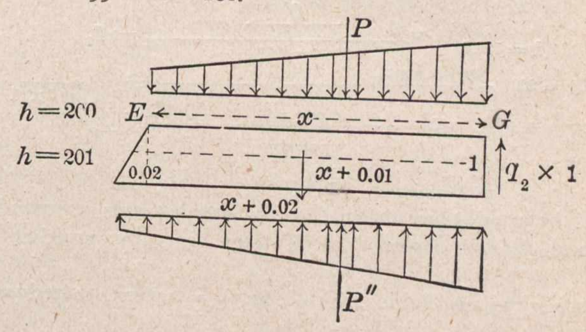

Fig. 3.

Consider a portion of the dam, Fig. 4, bounded by the water face; the plane, F M, at the level, $h = 199.5$, on which the total shear is Q′, the plane, E N, at the level 200.5, on which the total shear is q, and the vertical plane, M N, 1 sq. ft. in area, on which the average normal stress is p′. The water pressure on E F will be supposed to be exerted horizontally. It is equal to 80 units. Assuming, as stated, that $q_1 =$ intensity of horizontal shear at M, and $q_2 =$ the corresponding intensity at N, we have, taking the origin as before, O,

$$Q' = \int_{0.01}^{x} q_1\ dx; \qquad Q = \int_{-0.01}^{x} q_2\ dx;$$

or,

$$Q' = 0.006494794 - 0.6542906\ x + 0.481125\ x^2 - 0.00023\ x^3$$

$$Q = -0.00640186 - 0.6353780\ x + 0.480785\ x^2 - 0.0006803\ \frac{x^3}{3}$$

Checks.—The total water pressure for $h = 199.5$ is $\frac{1}{5}(199.5)^2 = 7960.05$ and for $h = 200.5$, $\frac{1}{5}(200.5)^2 = 8\ 040.05$. The first should equal Q′, for $x = 133.665$, or 7 959.22; the second should equal Q, for $x = 134.335$, or 8 041.12. The slight differences tend to give confidence in the results.

For equilibrium, the sum of the horizontal forces acting on E F M N, Fig. 4, must be zero;

therefore, $p' = 80 + Q - Q'$ .... (7)

$$p' = 80.01 - 0.0189\ x + 0.00034\ x^2 - 0.00000323\ x^3.$$

This average stress will now be assumed to be the intensity of the horizontal unit stress on vertical planes at $h = 200$.

It will now be perceived why a seven-place table was necessary in the computations, the coefficients of x^2 and x^3

* The writer desires here to acknowledge his indebtedness to a recent paper on "Stresses in Masonry Dams" by Ernest Prescot Hill, M. Inst. C. E., published in Minutes of Proceedings, Inst. C. E., Vol. CLXXII., p. 134. Mr. Hill considers the case of a dam with a vertical inner face. By the aid of the calculus, he effects an exact solution, which leads to general formulae for shear and normal pressures on vertical planes.

Mr. Hill ascribes to Professor W. C. Unwin the suggestion, "that the shearing stress at any point may be found by considering the difference between the total net vertical reactions [between that point and either face] along two horizontal planes at unit distance apart," and states that Prof. Unwin "has applied the principle to a triangular dam by the use of algebraical methods."