LEMMA II. "If a straight line cut the sides of a triangle, or the sides produced, the product of three segments in order is equal to the product of the other three segments. Let ABC be a triangle, and let a straight line be drawn cutting the side BC at D, the side CA at E, and the side AB produced through A at F. Then BD and DC are called *segments* of the side BC, and CE and EA are called *segments* of the side CA, and also AF and FB are called *segments* of the side AB. The product BD. CE. AF, is equal to the product DC. EA. FB." Todhunter's Euclid. Appendix, Prop. 55. (McDowell's Exercises in Euclid, Problem 167, Mulcahy's Modern Geometry, Sec. 9, Lem. 2, or in fact any work on the Transversals.)

LEMMA III. ABC is a double inclined plane, the base AC being horizontal. If two weights resting on these planes and so connected by an inextensible string passing over B, balance without friction, then shall the weights be as the lengths of the planes on which they rest.

Let the weight resting on AB be *W*, and that on CB be *w*. Resolve the weights along and perpendicular to the planes. The resolved parts along the planes, representing the tension of the string, must be equal, say to *T*. Draw BD perpendicular to AC.

$$W : T :: AB : BD$$
$$w : T :: CB : BD$$
$$\therefore W : w :: AB : CB.$$

PROBLEM.—Let A and B be any simultaneous positions of the weights, p the pulley over which the string passes, and P the point of intersection of the inclined planes and the plane through A, B, and p.

1° Since there is no friction the plane APB will be vertical.

2° p will be at P and may therefore be represented by it. Draw AX to represent the weight at A and from X drop XY perpendicular to AP. Let pA produced meet XY in Z, the tension on the string Ap will be represented by ZA. Now AY is constant as A moves up or down the plane; hence, by Lem. I, if p be outside the line of AP, ZA will increase as it moves up the plane and decrease as it moves down, but if p be in the line of AP, ZA will remain constant being in fact YA. Similarly it may be shown that as the weight at B moves down the plane the tension of the string will decrease if p be outside the line of BP, but will remain constant if p lie in that line Since the string is inextensible, as A moves up the plane, B moves down; hence, if p lie outside the lines AP, BP as the tension from A increases, that from B will decrease and *vice versa*; if p lie in one of the lines but not in the other, one tension will remain constant and the other will vary; but if p be in *both* the lines, *i.e.* at P, the tensions will remain constant and if equal at one position will be equal at all. But the problem requires the weights to balance in at least two positions; hence, p must be at P and the weight will balance in all positions.

3° When the weights are in the same horizontal lines, let their positions be C for *W* the weight at A, and D for *w* that at B. CD (which draw) is consequently a fixed horizontal line. Join AB intersecting CD in G.

Since $AP + PB = CP + PD$

$$AC = BD$$

Also $PD.W = PC.w$. (Lemma III.)

and $PC.AG.BD = PD.BG.AC$, (Lem. II.);

$\therefore AG.W = BG.w$

$\therefore$ G is the centre of gravity of *W* at A and *w* at B. But G lies in the fixed horizontal line CD, merely moving along it as *W* and *w* move. Hence the proposition.

Mr. Barnes, the proposer of this problem points out that particular cases of it have frequently been set at Cambridge. The usual restrictions are that one or that both the planes are perpendicular. (The latter gives the fixed pulley.) The editor finds the problem in Creswell's Maxima and Minima, Ap. II. The proposition is an immediate deduction from the law of conservation of energy.

(92). Most of our correspondents have not noticed that this is the general problem promised in the note to the solution of (87.)

Let G be the centre of gravity of the beam which need not be uniform or homogeneous. (Draw the figure with the beam projecting over the prop.) Let AD equal *a*, DC equal *b*, AG equal *c*, AC *m* ($\therefore m^2 = a^2 + b^2$) *W* equals weight of beam, *R* equals the reaction of the prop, and *T* equals the tension of the string.

Resolve *W* parallel and perpendicular to the beam putting *w* for the latter component; also resolve *R* which is perpendicular to the beam into vertical and horizontal components. The latter will equal *T* in magnitude.

Thus we get

$$W : w :: m : a$$
$$R : T :: m : b$$

Take moments around A,

$$cw = mR$$
$$\therefore T = \frac{abc}{m^3} W.$$

(93). Their rates will be as as products of their relative angular velocities into the lengths of the hands (the radii of the circles described by the extremeties,) *i.e.* as