$=0$, or substitute $a=b+c$, in each expression, and result is 0; $\therefore$ these expressions are completely divisible.

3. When is an expression completely symmetrical with respect to two or more letters? Apply the principle of symmetry to simplify the following:

$$(1)\ \frac{(a-b)^2-(b-c)^2}{a^2+ab-bc-c^2}+\ldots+\ldots$$

$$=\left(\frac{(a-c)\,(a-b+c)}{(a-c)\,(a+b+c)}\right)+\ldots+\ldots=0.$$

$$(2)\ \frac{(a-b)^2-c^2}{a^2-(b+c)^2}+\ldots+\ldots$$

$$=\frac{(a-b-c)\,(a-b+c)}{(a-b-c)\,(a+b+c)}+\ldots+\ldots=1.$$

4. Resolve the following expressions into factors, stating any general principles on which the method in each depends:

(1) $(a^2-b^2)^5+(b^2-c^2)^5+(c^2-a^2)^5$.

(2) $(m+1)x^3+(m+n)(x-y)xy-(n-1)y^3$.

(3) $ax^2-(2a+b)xy-(3a-c)xz+2by^2+(3b-2c)yz-3cz^2$.

4. We have where $a+b+c=0$,

$$(1)\ \frac{a^5+b^5+c^5}{5}=\frac{a^3+b^3+c^3}{3}\cdot\frac{a^2+b^2+c^2}{2}$$

applying this

$$(a^2-b^2)^5+.+.=5(a^2b^4-a^4b^2+\ldots)(a^4+b^4+c^4-2a^2b^2-2b^2c^2-2c^2a^2).$$

(2) $(m+1)x^3-(m+n)(x-y)xy-(n-1)y^3$
$=m(x^3-x^2y+y^2x)-n(y^3+x^2y-xy^2)+x^3+y^3$
$=mx(x^2-xy+y^2)-ny(y^2-xy+x^2)+(x+y)(x^2+y^2-xy)$
$=\{(m+1)x-(n-1)y\}\{x^2-xy+y^2\}$.

(3) Given expression
$=(ax-by+cz)(x-2y-3z)$.

5. Mention any artifices that may be used in solving equations involving fractions. Solve

$$(1)\ \frac{x+2m}{2n-x}+\frac{x-2m}{2n+x}=\frac{4mn}{4n^2-x^2}.$$

$$(2)\ \frac{3x-1}{x-1}-\frac{3x+9}{x+2}=\frac{28x-80}{7(x-3)}+\frac{28x+12}{7(x+4)}.$$

5. (1) $x=\dfrac{mn}{m+n}$.

6. Solve $ax+by=c$, $a'x+b'y=c'$, and interpret the results where

(i.) $\dfrac{a}{a'}=\dfrac{b}{b'}$. (ii.) $\dfrac{a}{a'}=\dfrac{b}{b'}=\dfrac{c}{c'}$.

6. See Todhunter's Larger Algebra, pp 118 and 119.

7. A student had four numbers in proportion, and thinking the numbers too large for convenience in working, he diminished each term of the proportion by the same number, and so obtained the result $41:93::7:51$. What was the true proportion?

7. We have $a:b::c:d$ suppose, and $a-x=41$, $b-x=93$, $c-x=7$, $d-x=51$. Whence $x=180$, and true proportion is
$221:273::187:231$.

8. Find the relations between the roots and coefficients of the quadratic

$$ax^2+bx+c=0.$$

If the roots of the equation $a^2x^2+bx+bc+b^2=0$ are equal, show that $\dfrac{1}{4a^2}-\dfrac{c}{b}=1$.

8. $ax^2+bx+c=0$ has equal roots if $b^2=4ac$; $\therefore$ in example there are equal roots if $4a^2(bc+b^2)=b^2$, or dividing both sides by $4a^2b^2$ and transposing if $\dfrac{1}{4a^2}-\dfrac{c}{b}=1$.

9. Solve

$$\text{i. } \frac{1}{x+a}+\frac{1}{x+b}+\frac{1}{x+c}+\frac{1}{x+a+b+c}=0.$$

ii. $x^4+x^3-4x^2+x+1=0$.

iii. $\dfrac{x^4+1}{(1+x)^4}=3\frac{1}{2}$. $\quad x+y+z=6$

iv. $x^2+y^2+z^2=14$. $\quad xy+yz=8$.

9 (i.) We get this equation to reduce to a cubic, finally to a quadratic, but have not space to give the solution; if any of our mathematical men will send us a short and easy solution we will be pleased to print it.

(ii.) Equation may be written

$$(x^4-2x^2+1)+x(x^2-2x+1)=0,$$

$$\text{or } (x-1)^2\{(x+1)^2+x\}=0,$$

$$\therefore x=1,\text{ or } \frac{3\pm\sqrt{5}}{2}.$$

(iii.) $\dfrac{1+x^4}{(1+x)^4}=3\frac{1}{2}$