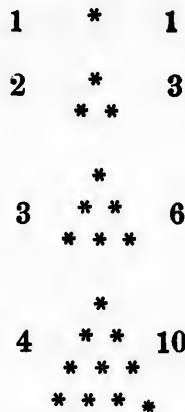


$$\frac{13(13+1)(2 \times 13+1)}{6} = \frac{13 \times 14 \times 27}{6}.$$

$$\therefore \text{whole number in the pile} = \frac{22 \times 23 \times 45 - 13 \times 14 \times 27}{6} = 2976.$$

### SERIES OF TRIANGULAR NUMBERS.

143. If objects be arranged in equilateral triangles upon a plane surface, the number required to form a complete triangle, as in the margin is called a triangular number. With 1 object upon a side we have 1 as the first triangular number. With 2 objects upon a side it requires 3 to complete the triangle; there being one row with one in it and a second row with two. With 3 upon a side we have 3 rows, of 1, 2 and 3 objects respectively; i.e., 6 in all. With 4 upon a side we have four rows of 1, 2, 3 and 4 objects, or 10 in all, &c.



Hence the series of triangular numbers is 1, 3, 6, 10, 15, 21, &c.

The numbers are evidently the successive sums of the series of natural numbers beginning at unity.

$$\text{Thus, } 1 = 1, 3 = 1 + 2, 6 = 1 + 2 + 3, 10 = 1 + 2 + 3 + 4 \\ 15 = 1 + 2 + 3 + 4 + 5, \text{ \&c.}$$

144. To find the sum of  $n$  terms of the triangular numbers.

Let  $\Sigma n$  denote the sum of  $n$  terms of the series of natural numbers,  $\Sigma n^2$  that of the series of square numbers, and  $\Sigma t$  the sum of  $n$  terms of the series of triangular numbers.

$$\text{Then, } \Sigma n = 1 + 2 + 3 + 4 \dots n,$$

$$\Sigma n^2 = 1 + 4 + 9 + 16 \dots n^2,$$

$$\Sigma n + \Sigma n^2 = 2 + 6 + 12 + 20 + \dots (n^2 + n).$$

$$= 2(1 + 3 + 6 + 10 + \dots \frac{n^2 + n}{2})$$