$$\frac{13(13+1)(2\times 13+1)}{6}=\frac{13\times 14\times 27}{6}.$$

$\therefore$ whole number in the pile $=\dfrac{22\times 23\times 45-13\times 14\times 27}{6}=2976.$

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## *SERIES OF TRIANGULAR NUMBERS.*

143. If objects be arranged in equilateral triangles upon a plane surface, the number required to form a complete triangle, as in the margin is called a triangular number. With 1 object upon a side we have 1 as the first triangular number. With 2 objects upon a side it requires 3 to complete the triangle; there being one row with one in it and a second row with two. With 3 upon a side we have 3 rows, of 1, 2 and 3 objects respectively; i.e., 6 in all. With 4 upon a side we have four rows of 1, 2, 3 and 4 objects, or 10 in all, &c.

Hence the series of triangular numbers is 1, 3, 6, 10, 15, 21, &c.

| | | |
|---|---|---|
| 1 | * | 1 |
| 2 | *<br>* * | 3 |
| 3 | *<br>* *<br>* * * | 6 |
| 4 | *<br>* *<br>* * *<br>* * * * | 10 |

The numbers are evidently the successive sums of the series of natural numbers beginning at unity.

Thus, $1=1$, $3=1+2$, $6=1+2+3$, $10=1+2+3+4$
$15=1+2+3+4+5$, &c.

144. To find the sum of $n$ terms of the triangular numbers.

Let $\Sigma n$ denote the sum of $n$ terms of the series of natural numbers, $\Sigma n^2$ that of the series of square numbers, and $\Sigma t$ the sum of $n$ terms of the series of triangular numbers.

Then, $\Sigma n=1+2+3+4\ldots\ldots n,$

$\Sigma n^2=1+4+9+16\ldots\ldots n^2,$

$\Sigma n+\Sigma n^2=2+6+12+20+\ldots\ldots(n^2+n).$

$=2(1+3+6+10+\ldots\ldots\frac{n^2+n}{2})$