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THE PHILOSOPHY OF HYPER-SPACE.*

THERE is a region of mathematical thought which might be called the fairyland of geometry. The geometer here disports himself in a way which, to the non-mathematical thinker, suggests the wild flight of an unbridled imagination rather than the sober sequence of mathematical demonstration. Imaginative he certainly does become, if we apply this term to every conception which lies outside of our human experience. Yet the results of the hypotheses introduced into this imaginary universe are traced out with all the rigor of geometric demonstration. It is quite fitting that one who finds the infinity of space in which our universe is situated too narrow for his use should, in his imaginative power, outdo the ordinary writer of fairy tales, when he evokes a universe sufficiently extended for his purposes.

The introduction of what is now very generally called hyper-space, especially space of more than three dimensions, into mathematics has proved a stumbling block to more than one able philosopher. The question whether a fourth dimension may possibly exist, and whether it can be legitimately employed for any mathematical purpose, is one on which clear ideas are not universal. I do not, however, confine the term 'hyper-space' to space of more than three dimensions. A hypothesis which is simpler in its fundamental basis, and yet seems absurd enough in itself, is that of what is sometimes, improperly I think, called curved space. This also we may call hyper-space, defining the latter in general as space in which the axioms of the Euclidean geometry are not true and complete. Curved space and space of four or more dimensions are completely distinct in their characteristics, and must, therefore, be treated separately.

The hypothesis of a fourth dimension can be introduced in so simple a way that it should give rise to no question or difficulty whatever. Indeed, the whole conception is so simple that I should hardly deem it necessary to explain the matter to a professional mathematical student. But as we all have to come in contact with educated men who have not had the time to completely master mathematical conceptions, and yet are interested in the fundamental philosophy of our subject, I have deemed it appropriate to present the question in what seems to me the simplest light.

The student of geometry begins his study with the theory of figures in a plane. In this field he reaches certain conclusions, among them that only one perpendicular can be drawn to a line at a given point, and that only one triangle can be erected with given sides on a given base in a given order. Having constructed this plane geometry, he passes to geomety of three dimensions. Here he enters a region in which some of the propositions of plane geometry cease to be true. An infinity of perpendiculars can now be drawn to a given line at a given point, and an infinity of triangles can be constructed on a given base with given sides. He has thus considered in succession geometry of two dimensions, and then passed to geometry of three dimensions. Why should he stop there?

*Address of the President before the American Mathematical Society, December 29, 1897.