ed by giving definite values to the surds in F, and taking the cognate functions without reference to these surds.

Proposition VIII

Let an equation of the m^{th} degree, whose coefficients are rational functions of a variable p, be, $X=0$; X having no rational factors; and let an algebraical root of this equation, in a simple integral form, arranged also so as to satisfy the conditions of Def. 8, be $f(p)$. Take u_1, a surd in $f(p)$, not a subordinate of any other surd in the function, with the index $\frac{1}{n}$; and let the cognate functions of $f(p)$, obtained by successively changing u_1, wherever it occurs in $f(p)$ in any of its powers, into u_1, $z_1 u_1$, $z_1^2\ u_1, \ldots\ldots, z_1^{n-1}\ u_1$, z_1 being an n^{th} root of unity, distinct from unity, be,

$$\phi_1 \text{ or } f(p),\ \phi_2,\ \phi_3, \ldots\ldots\ldots, \phi_n.$$

Let $F_1(x)$ denote the continued product of the terms, $x-\phi_1$, $x-\phi_2$, $\ldots\ldots\ldots$, $x-\phi_n$. The coefficients of the various powers of x in $F_1(x)$, made to satisfy the conditions of Def. 8, are (Cor. 3, Prop. VI.) clear of the surd u_1; and the terms, ϕ_1, $\phi_2, \ldots\ldots\ldots, \phi_n$, constitute (Prop. VII.) the series of the unequal cognate functions of $f(p)$, obtained by affixing definite values to all the surds in $F_1(x)$, [which are necessarily surds in $f(p)$], and taking the cognate functions without reference to the surd character of the surds so made definite. Should $F_1(x)$, which is clear of the surd u_1, not have the coefficients of the powers of x rational, let u_2, a surd in $F_1(x)$, not a subordinate of any other surd in $F_1(x)$, with the indev $\frac{1}{r}$, be successively replaced by u_2, $z_2 u_2$, $z_2^2\ u_2, \ldots\ldots\ldots, z_2^{r-1}\ u_2$; z_2 being an r^{th} root of unity, distinct from unity; and, in consequence of these alterations, let $F_1(x)$ become successively $F_1(x)$, ${}^2F_1(x)$, ${}^3F_1(x), \ldots\ldots\ldots, {}^rF_1(x)$; the functions which are ϕ_1, $\phi_2, \ldots\ldots\ldots, \phi_n$, in $F_1(x)$, becoming ϕ_{n+1}, ϕ_{n+2}, $\ldots\ldots, \phi_{2n}$, in ${}^2F_1(x)$, and becoming ϕ_{2n+1}, &c., in ${}^3F_1(x)$; and so on. Denote the continued product of the terms, $F_1(x)$, ${}^2F_1(x), \ldots\ldots$, ${}^rF_1(x)$, when the result is made to satisfy the conditions of Def. 8, by $F_2(x)$, which is (Cor. 3. Prop. VI.) an expression clear of the surd u_2, and such (Prop. VII.) that the functions, ϕ_1, $\phi_2, \ldots\ldots, \phi_{nr}$, [the