origin or point of relief and back to the gate, are reflected and transmitted again over the same course, as waves of rarefaction or sub-normal pressure. In this manner pressure waves, alternately super-normal and sub-normal, travel up and down the penstock until damped out by friction. During the time taken by the wave to traverse its course from gate to origin the pressure at the gate remains at its full value, either super-normal or sub-normal, as the case may be.

Briefly stated, then, the premises are: that when water flowing in a pipe is suddenly arrested, certain pressure waves, the characteristics of which are known, are produced and propagated along the pipe at constant speed and constant magnitude, and that the speed and magnitude of these waves may be calculated for any given conditions.

TABLE 1—BY ARITHMETIC INTEGRATION

Data: $L = 820$ ft. $V_0 = 11.75$ ft. per sec. $H_0 = 165$ ft. $T = 2.1$ sec. $a = 4,680$ ft. per sec. Friction neglected.

(1) Interval.	(2) Time, t.	(3) Gate, B.	(4) Head, H.	(5) Velocity, V.	(6) 145 Δ V, Δ h.	(7) Σ (Δ h), h_t.
0	0.0000	0.91476	165.00	11.75		
		0.03811		0.085	12.32	
¼	0.0875	0.87665	177.32	11.665		12.32
		0.03811		0.095	13.78	
½	0.1750	0.83854	191.10	11.570		26.10
		0.03811		0.10	14.50	
¾	0.2625	0.80043	205.60	11.47		40.60
		0.03811		0.112	16.20	
1	0.3500	0.76232	221.80	11.358		56.80
		0.03811		0.258	37.40	
1¼	0.4375	0.72421	234.56	11.10		69.56
		0.03811		0.29	42.10	
1½	0.5250	0.68610	249.10	10.81		84.10
		0.03811		0.30	43.50	
1¾	0.6125	0.64799	263.60	10.51		98.60
		0.03811		0.33	47.80	
2	0.7000	0.60988	279.00	10.18		114.00
		0.03811		0.43	62.30	
2¼	0.7875	0.57177	291.14	9.75		126.14
		0.03811		0.47	68.20	
2½	0.8750	0.53366	302.70	9.28		137.70
		0.03811		0.48	69.60	
2¾	0 9625	0.49555	314.30	8.80		149.30
		0.03811		0.53	76.80	
3	1.0500	0.45744	327.90	8.27		162.90
		0.03811		0.58	84.00	
3¼	1 1375	0.41933	337.46	7.69		172.46
		0.03811		0.61	88.40	
3½	1.2250	0.38111	346.10	7.08		181.10
		0.03811		0.62	90.00	
3¾	1.3125	0.34300	354.90	6.46		189.90
		0 03811		0.67	97.10	
4	1.4000	0.30489	361.60	5.79		196.60
		0.03811		0.68	98.60	
4¼	1.4875	0.26678	366.60	5.11		201.64
		0 03811		0.70	101.50	
4½	1.5750	0.22867	371.10	4.41		206.10
		0.03811		0.715	103.80	
4¾	1.6625	0.19056	376.10	3.695		211.10
		0 03811		0.730	106.00	
5	1.7500	0.15245	378.30	2.965		213.30
		0.03811		0.730	106.00	
5¼	1.8375	0.11434	380.66	2.235		215.66
		0.03811		0.745	108.00	
5½	1.9250	0.07623	382.70	1.490		217.70
		0.03811		0.745	108.00	
5¾	2.0125	0.03812	381.90	0 745		216.90
				0.745	108.00	
6	2.1000	0.0	381.70	0.0		216.70

Fundamental Equations

It is the writer's intention to apply herein this theory of pressure waves to the phenomena which occur when the gates of a turbine at the end of a penstock are gradually closed. The damping effect of friction on the pressure waves will be neglected during the time of closure. The variable velocity of the pressure wave due to the difference in density of the water at the top and bottom of the penstock will also be neglected. For the sake of simplicity, it will be assumed that the gate opening is closed uniformly from full open to shut by a governor which moves the gates at uniform velocity from the beginning of its stroke to the end, and that the area of gate opening is directly proportional to the amount of gate movement. It may be stated here, parenthetically, that the resulting formulas may be modified, easily, to suit any method of gate closure, whether the speed of closing and the relation of governor movement to area of gate opening is uniform or variable.

When the gates of a turbine are closed gradually the velocity of the water in the penstock is reduced to zero and the pressure in the penstock rises. It is frequently assumed that the reduction in velocity takes place uniformly, but the rise of pressure, which commences immediately after the gates begin to move, increases the velocity of discharge through the gate opening, and, during the early part of the gate movement, tends to diminish the rate at which the flow of the water is retarded. This variable rate of retardation, during the time the gates are being closed, has an important bearing on the resulting rise of pressure, and it is necessary to take it into consideration by determining the relation between the velocity of flow and the pressure in the penstock. This relation may be expressed by the equation:—

$$V_0 = B_0(H_0)^{1/2} \quad \ldots\ldots (3)$$

where V_0 = initial velocity, in feet per second, of the water in the penstock before shut down;

H_0 = normal net head, in feet;

and B_0 = a number representing the gate opening. The value of B_0 is best determined from the known value of V_0 and H_0, but, in fact, B_0 is the ratio of the area of the penstock multiplied by $(2g)^{1/2}$ times the coefficient of discharge of the gate opening.

At any time during the closing of the gates the relation between V, B, and H would be expressed by the general formula, $V = B(H)^{1/2}$. If the gates are closed in the time, T, and t is the time from the beginning of the stroke to any time before the end of the stroke, the value of B_t (that is, B at the end of the time, t) for uniform closing, would be $(1-t/T)B_0$. During the time, t, the pressure in the penstock has risen an amount, h_t, so that the net head, H_t (that is, H at the end of the time, t) would be equal to H_0+h_t. Therefore, the expression for the value of V_t (that is V at the end of the time, t) is:

$$V_t = (1-t/T)B_0(H_0+h_t)^{1/2} \quad \ldots\ldots (4)$$

TABLE 2—BY ARITHMETIC INTEGRATION

Data: $L = 6,337$ ft. V_0 15,055 ft. per sec. $H_0 = 1,260$ ft. $T = 69.5$ sec. $a = 3,647$ ft. per sec. Friction head $h_f = 81$ ft.

(1) Interval.	(2) Gate,* B.	(3) Head, H.	(4) Velocity, V	(5) 113 Δ V, Δ h.	(6) Σ Δ h.	(7) 3 574 V^2, h_f.	(8) Δ h_f.	(9) Σ(Δh + Δh_f) h_t.
0	0.4241	1260.0	15.055		....	81.0	....	....
			0.155	17.5				
1	0.4168	1279.2	14.90		17.5	79.3	1.7	19.2
			0.29	32.8				
2	0.4084	1279.8	14.61		15.3	76.5	4.5	19.8
			0.28	31.7				
3	0.3995	1284.0	14.33		16.4	73.4	7.6	24.0
			0.33	37.3				
4	0.3895	1291.8	14.00		20.9	70.1	10.9	31.8
			0.37	41.8				
5	0.3784	1295.5	13.63		20.9	66.4	14.6	35.5
			0.42	47.5				
6	0.3657	1305.3	13.21		26.6	62.3	18.7	45.3
			0.49	55.4				
7	0.3513	1311.8	12.72		28.6	58.0	23.0	51.8
			0.53	59.8				
8	0.3354	1319.0	12.19		31.0	53.0	28.0	59.0
			0.60	67.8				
9	0.3180	1329.8	11.59		36.8	48.0	33.0	69.8
			0.65	73.5				
10	0.2991	1334.9	10.94		36.7	42.8	38.2	74.9
			0.72	81.3				
11	0.2781	1348.2	10.22		44.6	37.4	43.6	88.2
			0.84	94.8				
12	0.2544	1359.7	9.38		50.2	31.5	49.5	99.7
			0.92	104.0				
13	0.2285	1369.2	8.46		53.8	25.6	55.4	109.2
			0.97	109.5				
14	0.2009	1376.6	7.49		55.7	20.1	60.9	116.6
			1.07	121.0				
15	0.1719	1391.6	6.42		65.3	14.7	66.3	131.6
			1.15	130.0				
16	0.1411	1395.8	5.27		64.7	9.9	71.1	135.8
			1.21	136.8				
17	0.1083	1407.2	4.06		72.1	5.9	75.1	147.2
			1.33	150.1				
18	0.0725	1416.3	2.73		78.0	2.7	78.3	156.3
			1.365	154.2				
19	0.0368	1416.5	1.365		76.2	0.7	80.3	156.5
			1.365	154.2				
20	0.	1419.0	0.		78.0	0.	81.0	159.0

* Non-uniform gate motion.

Calculation by Arithmetic Integration

Before proceeding with the analytical determination of h_t, it will perhaps make the work clearer to show first, by a numerical example, how h_t may be obtained by the trial-and-error method of arithmetic integration. Assume that the gate, instead of being moved in a continuous uniform manner, is closed by a series of small instantaneous move-