B3.0 DERIVATION OF $\frac{\partial V_e}{\partial e}$ FOR AN INITIALLY CIRCULAR ORBIT

$$\Delta V_e = \sqrt{\frac{2\mu}{a}\frac{(1+e)}{(2+e)}} - \sqrt{\frac{\mu}{a}} + \sqrt{\frac{2\mu}{a(1+e)(2+e)}} - \sqrt{\frac{\mu}{a}\frac{(1-e)}{(1+e)}} \qquad \text{(B3.1)}$$

Using series expansion, the following approximations are introduced.

$$\sqrt{1+e} \doteq 1 + e/2 \qquad \text{(B3.2)}$$

$$\sqrt{1-e} \doteq 1 - e/2 \qquad \text{(B3.3)}$$

$$\frac{1}{\sqrt{1+e}} \doteq 1 - e/2 \qquad \text{(B3.4)}$$

$$\frac{1}{\sqrt{2+e}} \doteq \frac{1}{\sqrt{2}}\left(1 - \frac{e}{4}\right) \qquad \text{(B3.5)}$$

Substituting Equations (B3.2) through (B3.5) into (B3.1) simplifying and eliminating second order terms in e gives

$$\Delta V_e \doteq \sqrt{\frac{\mu}{a}}\left(\frac{e}{2}\right)$$

Since the initial state is assumed to be e = o, therefore

$$e = \Delta e$$

and so, as $\Delta e \to o$

$$\boxed{\frac{\partial(\Delta V_e)}{\partial e} = \frac{1}{2}\sqrt{\frac{\mu}{a}}}$$