

is, for the most part, nothing else than arranging into such groups numbers that are given us either only partially or not at all so arranged. To take in illustration one example more. When the pupil is first asked to add thirty-six and seven, he virtually says, "I have three tens already grouped, but I have six and seven that are to be grouped. I will make the seven up to ten by taking three from the six and adding it to the seven. Then I shall have another group of ten that added to the three groups I had before will give four groups of ten, forty, and three more left from the six make up forty-three. Thirty-six and seven are forty-three." I do not say that the pupil uses such words as these or, indeed, any words at all in solving such an example. What I say is this that when rightly taught he goes through a process such as I have described, when first he discovers that thirty-six and seven are forty-three. After a time he has so frequently gone through the process of adding thirty-six to seven and other similar processes that he instinctively says, without consciousness of any process in the act, thirty-six and seven are forty-three. With a mind thus furnished and exercised a pupil is able to approach the multiplication table right.

A part of that table is already known to him from the intuitional arithmetic with which he is familiar. He sees as he glances at it mentally that four, which presents itself to him in such fashion as this 2×2 , is two twos 2×2 , that six 3×2 is two threes 2×3 or three twos 3×2 , that eight 4×2 is two fours 2×4 or four

twos 4×2 ; that ten 5×2 is two fives 2×5 or five twos 5×2 and that nine 3×3 is three threes 3×3 .

At this stage particular attention will be drawn to the commutative principle as applied to the product of two numbers, but without naming or enunciating the principle. It will suffice to point out that any product of two numbers may be stated in two ways. As we have seen that twice three and three times two are the same thing differently expressed, as also two fours and four twos and two fives and five twos, so we may see that any product written rectangularly on the black board may be read in two ways.

Thus.... may be indifferently read
as four lines of five dots
each or as five columns of
four dots each; that is four
 fives and five fours are identical.

A similar statement and demonstration may of course, and to the child obviously, be given respecting any other product of two numbers. This fact should be made quite familiar, as its ready use materially aids the pupil in rightly learning the multiplication table. One arrangement of the factors often presents their product to the understanding much more quickly than the other; three nines far more easily than nine threes are seen to be twenty-seven. Besides the number of products to be committed to memory is by this device halved; the work of learning the multiplication table looks to the child like a much less forbidding task when thus abbreviated.

The progressive teacher will keep steadily before him the purpose of the multiplication table. It is in short a regrouping of