

§14. If we wish to exhibit the root as in §9, we find from (8) that

$$B + B'\sqrt{z} = \frac{13}{100} \left(3 + \frac{7\sqrt{5}}{5} \right).$$

Therefore, since $a^5 z^2 \sqrt{z} = -\frac{\sqrt{5}}{625}$,

$$u_1^5 = \frac{13}{100} \left(3 + \frac{7\sqrt{5}}{5} \right) + \sqrt{\left\{ \left(\frac{13}{100} \right)^2 \left(3 + \frac{7\sqrt{5}}{5} \right)^2 + \frac{\sqrt{5}}{625^2} \right\}},$$

with corresponding expressions for u_2^5, u_3^5, u_4^5 . As a matter of fact,

$$\left(\frac{13}{100} \right)^2 \left(3 + \frac{7\sqrt{5}}{5} \right) + \frac{\sqrt{5}}{625^2} = \frac{1}{625^2} \left\{ \frac{47}{8} (21125 + 9439\sqrt{5}) \right\}.$$

§15. It is interesting to observe the application of the theory to the equation

$$x^5 - 3x^2 + 2x + 1 = 0, \quad (32)$$

whose roots, with the signs changed, are the same as the roots of the equation (27). By reference to §7, keeping in view that $k = -\frac{p_3}{20}$, it will be seen that, wherever an odd power of p_3 occurs as a factor in a term of any one of the coefficients of the equation $F(y) = 0$, an odd power of p_5 occurs as a factor of the same term. It follows that, by changing the signs of both p_3 and p_5 in the equation (27), in other words, by passing from the equation (27) to the equation (32), $F(y)$ remains unchanged. Therefore the commensurable root of this equation, which we have seen to be $\frac{1}{125}$, is the value of $a^2 z$ for the equation (32)

as well as for the equation (27). To find t or $\frac{c}{a}$, the equations (17) give us

$$yt = \frac{vn - 8kr}{vm - 8kq}.$$

The values of $vn - 8kr$ and $vm - 8kq$ given in §7 show that, in passing from (27) to (32), $vn - kr$ simply changes its sign, while $vm - kq$ remains unaltered.

Hence t or $\frac{c}{a}$ has the same absolute value for the equation (32) as for the equation (27), the signs, however, being different in the two cases. Consequently, for the equation (27), $\frac{c}{a} = -\frac{7}{4}$. Thus we get

$$a = -\frac{9439}{25 \times 4225}, \quad c = \frac{7 \times 9439}{422500}, \quad z = \frac{5 \times 4225^2}{9439^2}, \quad e = -\frac{398}{9439}.$$