stance, supposing that the longest train on the road will be 500 feet (engine and 14 cars), then

$$\frac{\frac{8}{10}\text{ per cent.} \times 100}{500} = \frac{1}{10}\text{ per cent. change}$$

per 100 feet will be the amount strictly demanded for complete safety on a road of the given length of train using link and pin couplers. But as automatic vertical plane couplers, with practically no slack, come more generally into use, which is only a question of a few years, the need for such extensive curves will not be imperative, and a vertical curve changing more rapidly will answer fully, when a longer curve is difficult to obtain. Usually, however, a level grade between the descending and ascending grades is required, because a structure should always be placed on a uniform grade from end to end, and as they are usually in the depressions, this limits the vertical curve in such cases to two short pieces joining the level grade to the others. (See Fig. 10.) If there is no break in the embankment a continuous vertical curve is much better from every point of view, and should be put in as in Fig. 11.

On roads having only light grades, and consequently heavier and longer trains, the rate of change in depressions will be very small, and circumstances will determine whether the full amount can be put in without excessive cost; but with light grades and easy vertical curves, the distance which the middle of the curve will rise above the point of intersection is small. It may be calculated, in any case, in the same manner as the middle distance in horizontal circular curves, if the vertical curve is treated as a circle, or if treated more precisely as a parabola, it may be stated at once as half the distance which the apex is from the middle of a cord drawn from one end of the vertical curve to the other, this being a fundamental equation of the parabola.

(*a*) Treating the vertical curve as an arc of a circle, calculate first the permissible change in grade per 100 feet divide this into the total change of grade, giving the total length of curve, ½ of which will be on each side of the apex of grades; then the position of the curve for each 100 feet relatively to the tangent lines may be obtained graphically on a large scaled drawing, or calculated more precisely as in ordinary horizontal circular curves.

(*b*) Treating the vertical curve as a parabola having a constant rate of change of direction per 100 feet, is more precise and more convenient. Calculate first the length of curve, which will be the same as in (*a*), and then proceed as follows: Let the change of grade per 100 feet = R. Then referring to figure 12—, the departure of the curve from the tangent will be ½R, 2R, 4½R, 8R, 12½R, ... etc., till the middle of the curve is reached, after which the distances from the second tangent will recede . . . . 12½R, 8R, 4½R, 2R, ½R, to the other end. It will be seen that by the latter method the elevations are always in even units or portions of units, and the rise of the curve above the tangents is given almost by inspection; for convenience the length of a vertical curve should be fixed at the nearest *even* hundred feet, so that the curve may be divided into two equal parts of exact hundreds in length. Such vertical curves, with their elevations once established, will be no more difficult to place on the ground and build to than a succession of straight lines with abrupt changes in grade, and will give a track safer in depressions, having better drainage in summit cuts, and better in every respect, but increasing the cost of the road-bed slightly.

Art. 9.—Horizontal Circular Curves.

It is not necessary to treat here of the mathematics of the circle. There are several engineering field books which have considerable space devoted to methods of placing curves on the ground under ordinary or exceptional circumstances. Some of these books also contain, in addition to ordinary mathematical tables, tables of external secants and of sub-tangents for each degree of curve, and for each interval of one minute in the total intersection angle; these books are great time-savers in field operations, and should always be used.

In placing curves on the ground, it is preferable to establish the two tangents first, intersect them and measure in the BC and EC from the intersection or apex; then the curve can be run in from either or both ends and any error minimized. With very long flat curves on unstable ground, it may even be preferable to fix the middle of curve from the apex by measuring in the external secant, and then run the curve in from the ends and middle; the method sometimes adopted of running a curve in from the BC, and deflecting on to the second tangent at the EC, is very liable to establish it erroneously.

Another very important point is the method of keeping curve notes. The vernier should always read half of the total deflection of the curve from the BC up to the point on the curve toward which the telescope is pointing; this is a constant index of the position of any point. This method necessitates loosening the vernier-plate at each set-up and re-setting it to read the index reading of the back-sight; but it has the all important feature of enabling a transit to be set up at any point on a curve, and being sighted to any other point with a certain knowledge of what the vernier reading should be. Curves can be run in backward as easily as forward. Any other method of keeping notes will be found, in the end, less reliable and convenient. Whenever curves are sharper than 4° or 5° it is better to put in stakes every 50 feet even on easy ground, as the difference between the length of chord and curve for 100 feet measurements would be considerable; it is also convenient for cross-sectioning. In running in sharp curves, particularly curves having a large intersection angle, the greatest care is necessary in the chaining; poor results in checking up at the EC are usually traceable to the errors in measuring the subtangents or the curve itself.

It is often necessary to replace stakes that have been lost, or to put in intermediate stakes on curves without the aid of a transit; whenever this is the case it is valuable to remember the following formula, which is approximately true for all curves usually used on steam railways:

It is $O = .218\ N^2D$ . . . . (11)

Where O = offset from middle of a chord to the curve (in feet)

N = length of chord in 100 feet.

D = degree of curve.

Or, if simpler, remember that the offset from the middle of a 100-feet chord on a 1° curve is .22 feet, and that

(1) Offsets vary directly as the degree of curve.

(2) Offsets vary as the square of the length of chord, which is true up to 200 or 300 feet chords.

(3) Offsets, inward to a curve, from a prolonged chord are 8 times the offsets from the middle of the same length of chord outward to the same curve. This is illustrated in Fig. 13.

Circular curves are in general use on railways, but there have been isolated attempts at using the parabola, which have not been found satisfactory. The idea involved in its use was to have a curve of easy radius at the ends and sharper in the middle, but the train did not travel steadily, being in a constant state of change from beginning to end of curve. It has been found from the very