Per question

$$x_1^{x_3} = x_4,\ x_2^{x_4} = x_3,\ \therefore\ x_1^{x_3 x_4} = x_3.$$

&c=&c;

$$\therefore x_1^{x_3 x_4 \ldots x_n x_1 x_2} = x_1.$$

$$\therefore x_1 x_2 x_3 \ldots x_n = 1.$$

xi. Shew that

$$a^2 \cos 2(B-C) = b^2 \cos 2B + c^2 \cos 2C + 2bc \cos(B-C).$$

Using the equations $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$,

this becomes

$$\sin^2 A \cos 2(B-C) = \sin^2 B \cos 2B + \sin^2 C \cos 2C + 2 \sin B \sin C \cos(B-C).$$

or $(1 - \cos 2A) \cos 2(B-C)$

$$=(1-\cos 2B)\cos 2B + (1-\cos 2C)\cos 2C + \sin 2B \sin 2C + (1-\cos 2B)(1 \cos 2C);$$

writing $\cos 2(B+C)$ for $\cos 2A$, and simplifying, this becomes

$$\cos 2(B-C) \cos 2(B+C) = \cos^2 2B - \sin^2 2C,$$

a well-known identity.

xii. If D, E, F are the points of contact of the inscribed circle with the sides BC, CA, AB respectively, shew that if the squares of AD, BE, CF are in arithmetical progression, then the sides of the triangle are in harmonical progression.

$$AD = s - a \text{ \&c=\&c.}$$

The relation

$$(s-a)^2 + (s-c)^2 = 2(s-b)^2$$

when simplified becomes

$$2ac = ab + bc$$

or $$b = \frac{2ac}{a+c}.$$

$\therefore$ a, b, c are in harmonical progression.

Solutions by FRANK BOULTBEE, U.C. (*See* October number.)

16. If $a^2x^2 + b^2y^2 + c^2z^2 = 0$, (1)

$a^2x^3 + b^2y^3 + c^2z^3 = 0$, (2)

and $\frac{1}{x} - a^2 = \frac{1}{y} - b^2 = \frac{1}{z} - c^2$, (3)

prove that $a^4x^3 + b^4y^3 + c^4z^3 = 0$,

and $a^6x^3 + b^6y^3 + c^6z^3 = a^4x^2 + b^4y^2 + c^4z^2$.

Multiply a^2x^3 by $\frac{1}{x} - a^2$,

" b^2y^3 by $\frac{1}{y} - b^2$,

&c., &c.

and we get

$$a^2x^2 - a^4x^3 + b^2y^2 - b^4y^3 + c^2z^2 - c^4z^3 = 0,$$

but $$a^2x^2 + b^2y^2 + c^2z^2 = 0,$$

$$\therefore a^4x^3 + b^4y^3 + c^4z^3 = 0, \quad (4)$$

Treating (4) by (3) as we did (2) by (3), we get

$$a^6x^3 + b^6y^3 + c^6z^3 = a^4x^2 + b^4y^2 + c^4z^2.$$

18. If A, B, C are the angles of a triangle $\cos A + \cos B + \cos C$ is greater than 1 and not greater than $\frac{3}{2}$.

$$\cos A + \cos B + \cos C = 1 + 4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}.$$

Now $4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$ is positive,

$$\therefore \cos A + \cos B + \cos C > 1.$$

To prove

$$\text{Sin}\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2} \text{ not } > \tfrac{1}{8}.$$

If the triangle is equilateral, we have

$$\text{Sin}\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2} = \tfrac{1}{8}.$$

And it is easily seen that if we alter the angles of the triangle we always get smaller values for $\sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$,

$$\therefore \text{Cos}\, A + \cos B + \cos C \text{ not } > \tfrac{3}{2}.$$

Solution by ANGUS MACMURCHY.

17. If $a_1, a_2, \ldots a_n$ be n real quantities and $(a_1^2 + a_2^2 + \ldots + a_{n-1}^2)(a_2^2 + a_3^2 + \ldots + a_n^2)$ be equal to $(a_1a_2 + a_2a_3 + \ldots + a_{n-1}a_n)^2$, then $a_1, a_2, \ldots a_n$ are in geometrical progression.

Simplifying

$$(a_2^4 + a_1^2a_3^2 - 2a_1a_2^2a_3) + \ldots = 0$$

i.e. $$(a_2^2 - a_1a_3)^2 + \ldots = 0$$

$\therefore$ $a_2^2 = a_1a_3$, $a_3^2 = a_2a_4$, &c=&c.