

going from Q to R, therefore B must have taken $\frac{PR}{QR}$ times 4 hrs. to go from Q to R, and consequently to go from P to R which is $\frac{PR}{QR}$ times as far, B would require $\frac{PR}{QR} \times \frac{PR}{QR}$ 4 hrs. But it takes 9 hrs. for B to go from P to R,

$$\therefore \frac{PR}{QR} \times \frac{PR}{QR} \times 4 = 9$$

$\therefore QR$ is $\frac{2}{3} PR$, and PR and QR are together equal to 30 miles, $\therefore PQ$ is 6 miles.

PROBLEMS.

26. "From a given point, in the circumference of a given circle, to draw a straight line which shall touch the circle, by the help only of the *parallel ruler*."

27. To construct an equilateral triangle equal in area to a given triangle.

28. Sum the series

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \&c. + \frac{1}{n(n+1)(n+2)}$$

29. Find the equation of the fourth degree of which one root is $\frac{1}{2}\sqrt{3} + \sqrt{-1}$.

30. (a) In reducing $\frac{1}{19}$ to a decimal, we have

$$\begin{array}{r} 19 \overline{) 100} \quad .05 \\ \underline{95} \\ 5 \end{array}$$

Show how to find the required result without dividing any further by 19.

(b) Infer from this result the decimal corresponding to any other proper fraction having 19 for denominator.

31. Show that the equation

$$(b^2 - c^2)x^2 + (c^2 - a^2)y^2 + (a^2 - b^2)z^2 + 2c(b-a)xy + 2a(c-b)yz + 2b(a-c)zx = 0$$

is satisfied by the simultaneous conditions

$$\begin{cases} ax + by + cz = 0 \\ a + b + c = 0 \end{cases}$$

32. There are n persons possessing certain sums of money. The first of them gives to each of the others a sum equal to what each already has; then the second does the same; then the third, and so on to the n th. Show that any one of them—say the r th—will have gained or lost according as the sum he originally had is greater or less than

$$\frac{2^{n-r}}{2^n - 1} s$$

when s is the total sum held by all.

33. Show that the equation

$$\frac{2}{\sqrt{(x-a)}} + \frac{1}{\sqrt{(x-b)}} = \frac{1}{\sqrt{(x-c)}}$$

is reduced to a quadratic if

$$\frac{2}{\sqrt{a}} + \frac{1}{\sqrt{b}} = \frac{1}{\sqrt{c}}$$

34. A wine-merchant mixes two kinds of wine and sells the mixture so as to gain 8 per cent. on what the wine cost him. Had he sold each kind of wine at the same price per gallon as he sells the mixture, he would have gained 10 per cent. and 6 per cent. respectively on their cost price. In what proportion were the two kinds of wine mixed together?

TRIGONOMETRY.

Toronto University, First Year, 1876.

1. State and prove the rule for finding the characteristic of the logarithms of whole numbers.

Given $\log .25 = -.60206$ find how many digits there will be in the integral part of $(2.5)^{22}$.

2. Prove $\log(a^x) = x \log a$, $\log \frac{a}{b} = \log a - \log b$,

Evaluate the following by using logarithms:

$$\sqrt[4]{80} \times \sqrt[3]{27} \cdot \sqrt[5]{-5} \times 18^{-\frac{1}{5}}$$