representing two parallel (or, it may be, coincident) straight lines; the former reduces it to

$$By^2 + 2\,Dx = F,$$

and by taking the origin on the curve, still further to

$$y^2 = Lx,$$

representing the parabola.

3. *On a method of Approximating to the Square Root of a Number:*

The following singular proposition is given by Murphy in his *Theory of Equations*, Art. 77, and is very characteristic of a mathematician, perhaps, the most original of modern times. The demonstration that follows is his own, somewhat simplified. Let N be the number, and let $\sqrt{N}$ be between n and $n + 1$. Put $N - n^2 = a$, $(n + 1)^2 - n^2$, or, $(2\,n + 1) = b$. Take any proper fraction $\frac{u_0}{v_0}$, and let a series of fractions be successively formed by the law

$$u_{x+1} = av_x + u_x,\ v_{x+1} = bv_x + u_x,$$

then $\frac{u_x}{v_x}$ converges to the decimal part of $\sqrt{N}$.

For, $\frac{u_{x+1}}{v_{x+1}} = \frac{av_x + u_x}{bv_x + u_x}$, and is a proper fraction since $a < b$,

$$= \frac{a + \frac{u_x}{v_x}}{b + \frac{u_x}{v_x}}$$

Let then $y = \text{Limit}\ \frac{u_x}{v_x} = \text{Limit}\ \frac{u_{x+1}}{v_{x+1}}$;

then ultimately $y = \frac{a + y}{b + y}$

$$\text{or, } y^2 + (b - 1)\,y = a$$

$$\text{and } y^2 + 2ny + n^2 = N.$$

$$\text{whence } y = -\,n + \sqrt{N},$$

since the positive sign must be taken.

$$\text{Hence. Limit } \frac{u_x}{v_x} = \sqrt{N} - n,$$

or $\frac{u_x}{v_x}$ converges to the decimal part of $\sqrt{N}$.

Murphy gives as an example $\sqrt{10}$. Assume the fraction $\frac{1}{6}$; then $a = 1$, $b = 7$, and the successive convergents are

$$\frac{1}{6}, \frac{7}{43}, \frac{25}{154}, \frac{179}{1103}, \frac{1282}{7900}, \cdots\cdots\cdots$$