

$$\therefore \frac{x^2+1}{a^2+1} = \frac{2}{(1-a)^2} \quad (2)$$

and the prod. of (1) and (2) = $\frac{1-b}{1-a}$

$$\text{also, } \frac{x+1}{y+1} = \frac{1-b}{1-a}$$

hence the expression = 0.

$$\begin{array}{r} 4. \quad -1 \overline{) 7+21+35+35+21+7} \\ \quad \underline{-7-14-21-14-7} \\ \quad \quad -1 \overline{) 7+14+21+14+7} \\ \quad \quad \quad \underline{-7-7-7} \\ \quad \quad \quad \quad -1 \overline{) -7-7-7} \\ \quad \quad \quad \quad \quad \underline{-7-7-7} \\ \quad \quad \quad \quad \quad \quad 7+7+7 \end{array}$$

$\therefore$ Quotient is $7x^2 + 7xy + 7y^2$.

$$\begin{array}{r} -12 \overline{) 1+5-88-40+0-151} \\ \quad \underline{-12+84+48-96+1152} \\ \quad \quad 1-7-4+8-96, 1001 \end{array}$$

The value is $\therefore 1001$.

$$\begin{aligned} 5. \quad & 18x^4 + 9x^3 - 17x^2 - 4x + 4 \\ & = (3x+2)(3x-2)(2x-1)(x+1); \\ & \quad 8x^4 + 4x^3 - 6x^2 - x + 1 \\ & = (2x+1)(2x-1)(2x-1)(x+1). \end{aligned}$$

$$6. \quad \text{The first fraction should be } \frac{3}{4(1-x)^2}$$

and the expression reduces to

$$\frac{x^2+x+1}{(x^4-1)(x-1)}$$

7. If $a=1$, the equality holds for all values of the quantities involved, and is therefore an identity.

$$8. \quad \frac{ab}{a+b}, \frac{ac}{a+c}, \frac{bc}{b+c}$$

will be in H. P. if their reciprocals are in A. P., that is, if

$$(1) \quad \frac{a+b}{ab} - \frac{a+c}{ac} = \frac{a+c}{ac} - \frac{b+c}{bc}$$

$$\text{if } \frac{1}{b} + \frac{1}{a} - \frac{1}{c} = \frac{1}{a} + \frac{1}{c} - \frac{1}{b}$$

$$\text{if } \frac{1}{b} - \frac{1}{c} = \frac{1}{a} - \frac{1}{b}$$

i. e., if a, b, c , are in H. P.

$$(2) \quad \frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}, \text{ are in A. P.}$$

$$\text{if } 2 \frac{c+a}{b} = \frac{b+c}{a} + \frac{a+b}{c}$$

$$\text{if } 2 \frac{c+a}{b} + 2 = \frac{b+c}{a} + 1 + \frac{a+b}{c} + 1$$

$$\text{if } 2 \frac{a+b+c}{b} = \frac{a+b+c}{a} + \frac{a+b+c}{c}$$

$$\text{if } \frac{2}{b} = \frac{1}{a} + \frac{1}{c},$$

i. e., if a, b, c , are in H. P.

9. Let c be the constant, connecting the area of each circle with the square of its radius; then the areas of the first two circles are each $9c$, and of the others $16c, 25c, 36c, 49c$ respectively. The sum of these is $144c$, which is the area of a circle of radius 12.

10. If 11 is the first term, and d the common difference,

$$\begin{aligned} & \text{the sum of three terms} = 33 + 3d, \\ & \text{and sum of nine terms} = 99 + 36d; \end{aligned}$$

equating these we get $d = -2$.

$\therefore$ the series is 11, 9, 7, 5, 3, 1, -1, -3, &c.

11. Let r be the common ratio; then the fifth term is $3r^4$,

$$\therefore r^4 = \frac{16}{81} \text{ and } r = \frac{2}{3} \text{ or } -\frac{2}{3}$$

The second value of r gives the series required.

$$12. \quad \sqrt{1.77} = \sqrt{\frac{16}{9}} = \frac{4}{3} = 1.33.$$

13. (1) The sum of $2n$ terms

$$= (2a + 2nd) \frac{n}{2}$$

$$\text{sum of } n \text{ terms} = (2a + nd) \frac{n}{2}$$

and the sum of the latter half of $2n$ terms is equal to the difference between these,

$$\therefore = (2a + 3nd) \frac{n}{2}$$

The sum of $3n$ terms of this series

$$= (2a + 3nd) \frac{3n}{2}$$

(2) First take n terms in each series;